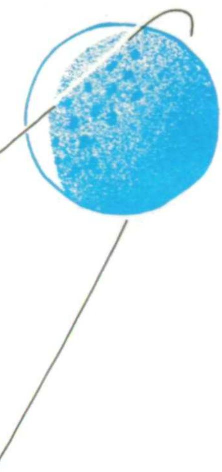

LOW-THRUST SOLAR ELECTRIC PROPULSION NAVIGATION SIMULATION PROGRAM

BY

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AMRL 1056 AUGUST 1973

APPLIED MECHANICS RESEARCH LABORATORY
THE UNIVERSITY OF TEXAS AT AUSTIN AUSTIN, TEXAS

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LOW-THRUST SOLAR ELECTRIC PROPULSION
NAVIGATION SIMULATION PROGRAM*

by

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General

Program LOGO is an interplanetary low-thrust, solar electric propulsion mission simulation program suitable for basic navigation feasibility studies. It employs simple, two-body dynamics to simulate the heliocentric phase (no n-body perturbations). Provisions are made to simulate uncertainties in the thrust program in a realistic manner, and to assess the effects of these uncertainties on various navigation strategies. One key feature is the ability to configure the dynamic model equations in a number of different ways to account for the thrusting uncertainties. Several navigation data types may be simulated: Earth-based radar range and range-rate, and on-board celestial observations involving the sun, Earth, and a specified navigation star. Although gravitational perturbations of the Earth acting on the spacecraft are not considered, rotational dynamics of the Earth are modeled to account for the significant effects of tracking station motion.

Several types of information output are available. These include detailed numerical information print output, and both printer plot and digital plot features. Further, a limited on-line display capability is available for teletype and CRT terminal facilities.

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MATHEMATICAL MODELS

Trajectory Simulation

The motion of the solar electric propulsion (SEP) spacecraft is assumed to be governed by the gravitational attraction of the sun (which is assumed to be perfectly known), and the thrust acceleration of the solar electric propulsion system. Further, random errors in the thrust program are assumed to influence the spacecraft motion. If only the central force attraction of the sun is included, the equations of motion for the SEP spacecraft are

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + T \tag{1}$$

where, as shown in Figure 1, r is a 3-vector of heliocentric position components, X, Y, Z; v is a 3-vector of heliocentric velocity components $\dot{X}$, $\dot{Y}$, $\dot{Z}$; $|r|$ is the magnitude of r; and μ is the gravitational parameter of the sun. T is the heliocentric thrust acceleration vector composed of the design thrust acceleration, T^*, as well as thrust acceleration errors from a number of sources (beam voltage and current, grid warpage, deadband control errors, etc.). The heliocentric components of T, $[T_X \vdots T_Y \vdots T_Z]$, may be expressed in a vehicle centred frame as $[T_x \vdots T_y \vdots T_z]$, where the two vectors are related by

$$T = \begin{bmatrix} T_X \\ T_Y \\ T_Z \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = R \begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} \tag{2}$$

where ψ is the heliocentric orientation angle (see Figure 1).

Random errors are assumed to occur in both the magnitude of the nominal thrust acceleration, a^*, as well as in the thrust vector pointing angles, γ and θ (see Figure 1). Thus, the true thrust acceleration vector, T, differs from the nominal thrust acceleration vector, T^*, in both magnitude and orientation. The true thrust acceleration, expressed in the vehicle centered reference frame (x, y, z) is (see Figure 1)

$$\begin{bmatrix} T_x \\ T_y \\ T_z \end{bmatrix} = a \begin{bmatrix} \sin\gamma \quad \cos\theta \\ \cos\gamma \\ \sin\gamma \quad \sin\theta \end{bmatrix} , \tag{3}$$

where $a = a^* + \delta a$. The acceleration error magnitude is simulated by $\delta a = \delta a_o \sin\omega t + u_a$ where δa_o and ω are constants and where the random variable, u_a, has the statistics $E\{u_a\} = 0$, $E\{u_a^2\} = \sigma_a^2$. For the design mission, T^* is assumed to be of constant magnitude along the vehicle centered y-axis and the nominal values of the pointing angles, γ and θ, are assumed to be related as shown in Figure 2.a. The radius of the circle is the maximum deviation, $\sin\bar{\gamma} \approx \bar{\gamma}$, of the thrust vector from the nominal position; it represents deadband pointing errors. The quantity $d \doteq s(t-t_b)$ is the distance of the tip of the thrust vector from the point where it last touched the boundary, $\bar{\gamma}$. The rate, $\dot{s}$, is simulated as a constant plus an additive noise component; t is the current mission time, and t_b is the time at which the boundary was last encountered. The angle, ϕ, is sampled from a uniform distribution U(-.0708, .866). In addition, purely random components are added to $\dot{s}$, γ, and θ. With these assumptions, Eqs. (1) can be integrated to obtain the simulated true trajectory. (Figure 3 shows the trace of the tip of the thrust acceleration vector as projected onto the local x-z plane.)

Models for Error Compensation

In the subsequent discussion, it is assumed that the thrust acceleration can be separated into modeled and error components, i.e., $T = T^* + m(t)$ where m(t) is a 3-vector of thrust acceleration error components. The errors, m(t),

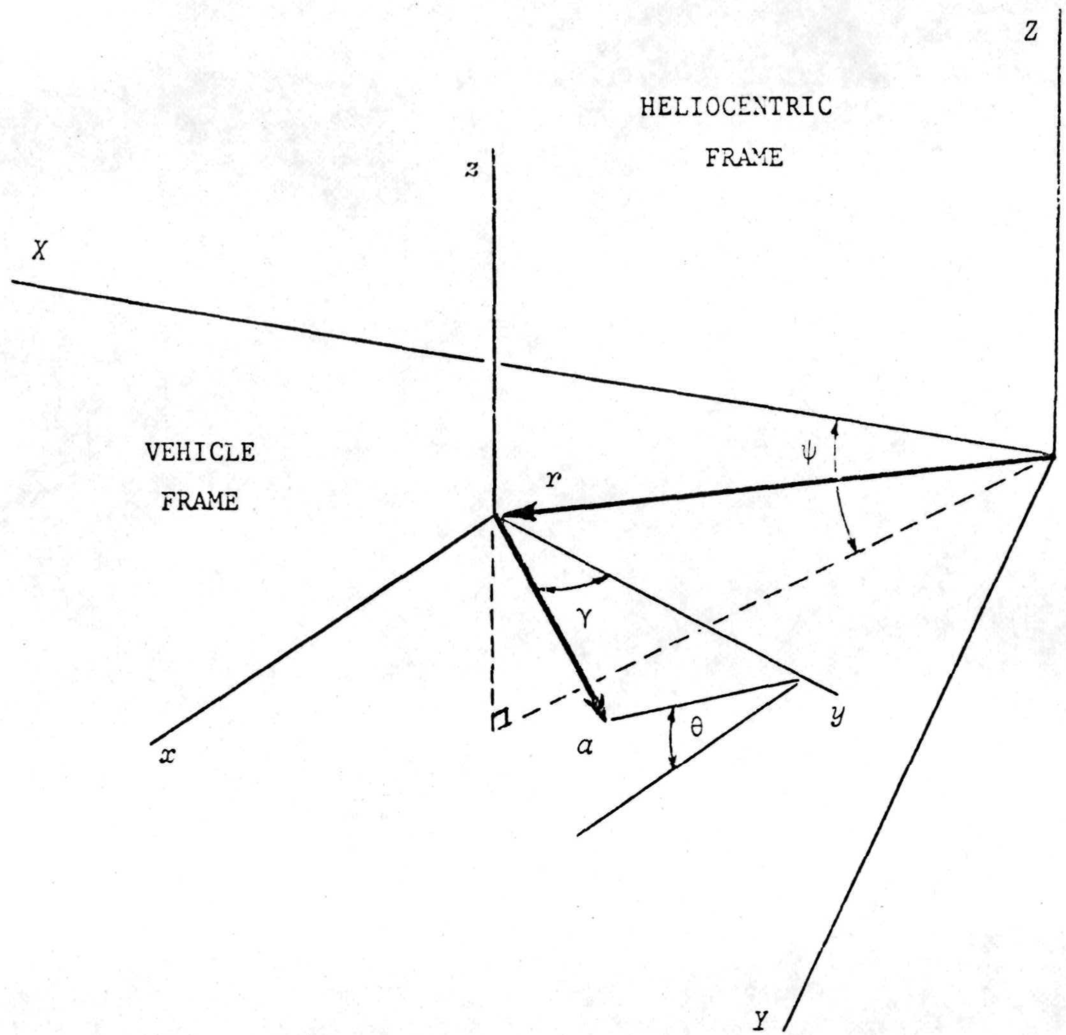

Figure 1. Reference Frames

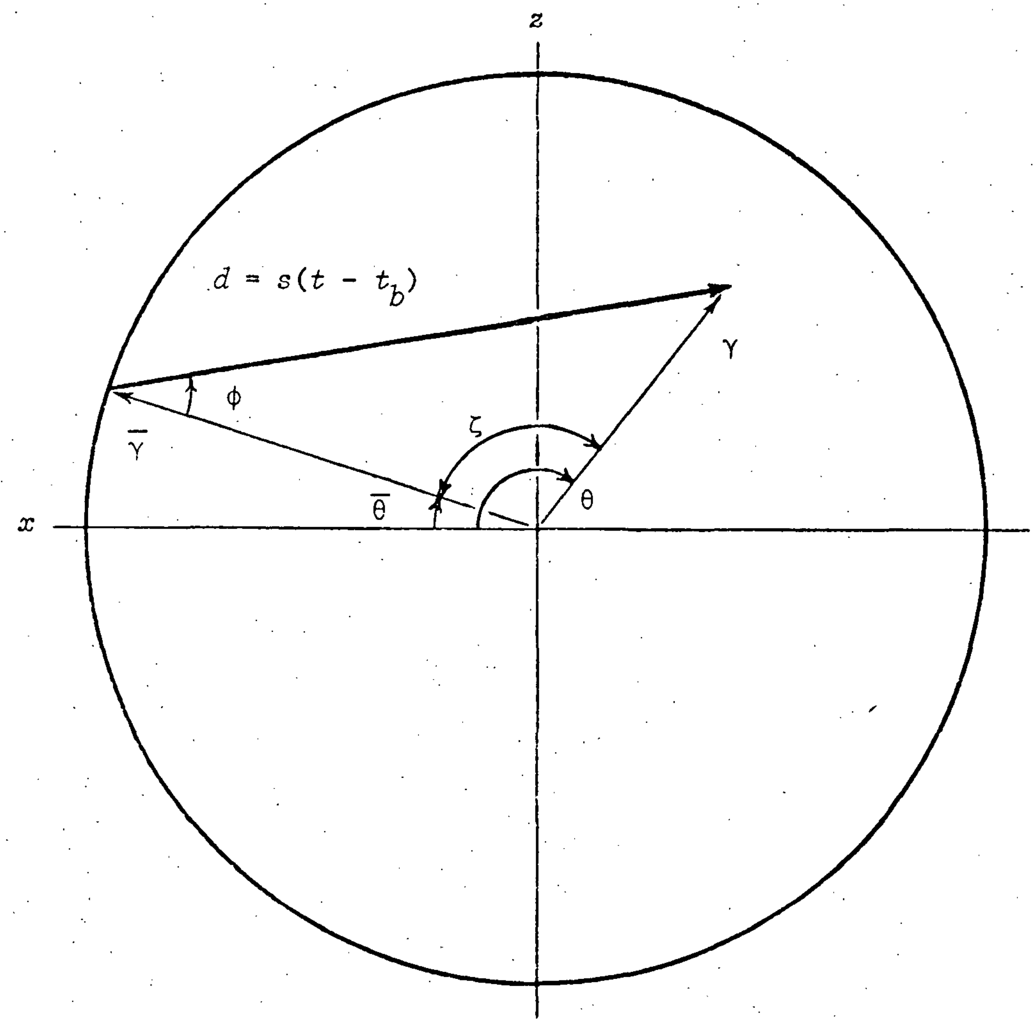

Figure 2.a. x - z Acceleration Error Components

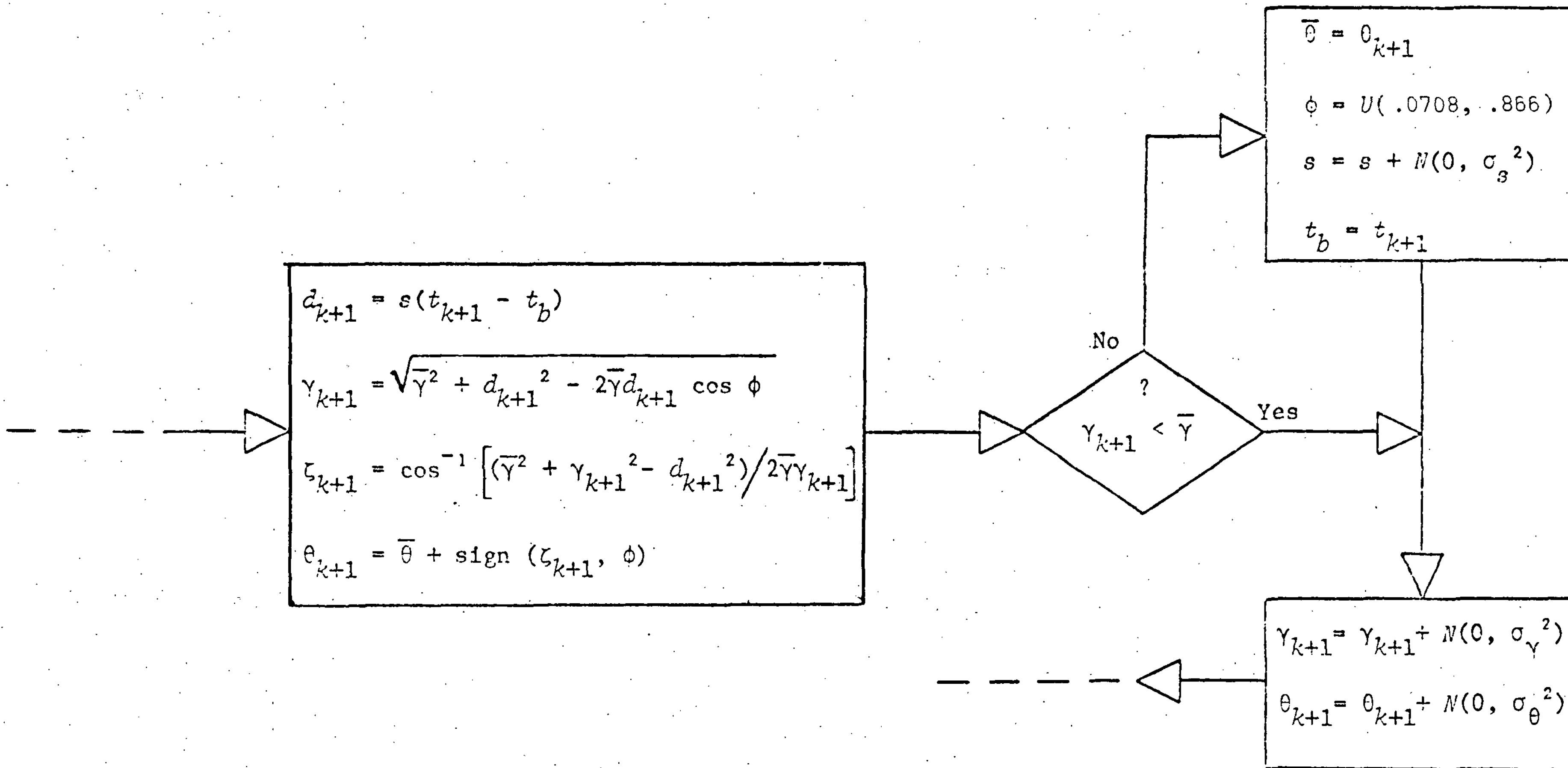

Figure 2.b. **Pointing Angles Simulation**

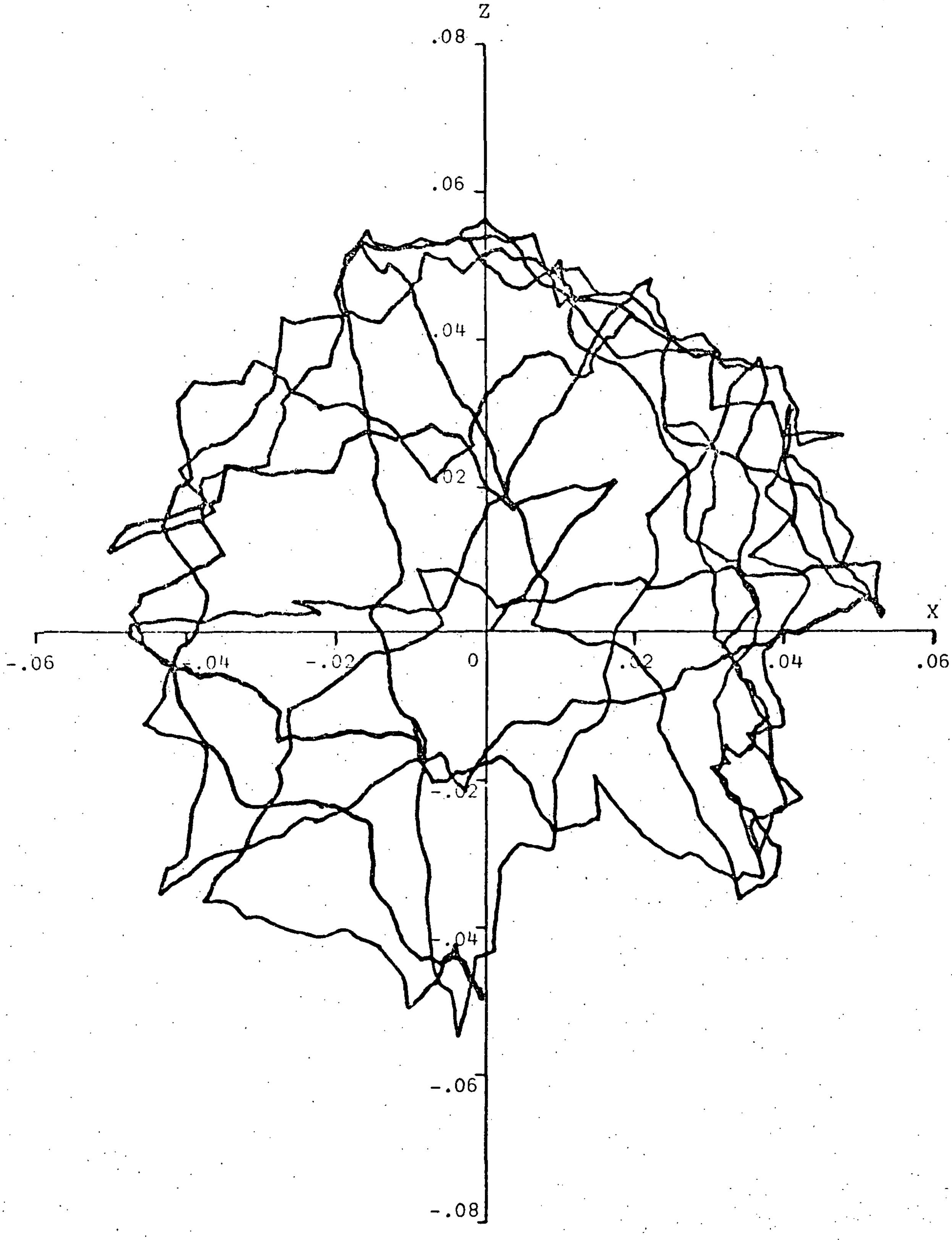

Figure 3. Acceleration Errors in x-z Plane
(10^{-4} m/sec^2)

are approximated by $\varepsilon(t)$ where $\varepsilon(t)$ satisfies one of several possible first-order or second-order differential equations. The values of $\varepsilon(t)$ and any unspecified parameters in the differential equations which describe $\varepsilon(t)$ are estimated simultaneously with the position and velocity components. Six models are available for use as approximations for $m(t)$. See Tables 1a and 1b.

Model 0. In this model an arbitrary state noise covariance matrix, Q, is added to the differential equation governing the state error covariance. The Q-matrix compensates for the process noise and is used to maintain a positive definite error covariance matrix. The differential equations for the state vector, $X^T = [r^T \; v^T]$, are

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + T^* + u \tag{4}$$

where u is a random 3-vector with the a priori statistics $E\{u\} = 0$, and $E[u(t)u^T(\tau)] = q(t)\delta(t-\tau)$. This model corresponds to a value of MODOP = 0 or MODOP = 1. (MODOP = 1 not fully implemented, do not use). See Table 1b for the form of state noise covariance logic.

Model 1. The thrust acceleration error components are approximated by a first-order Gauss-Markov process. The differential equations for the state vector, $X^T = [r^T \; v^T \; \varepsilon^T \; \eta^T \; \alpha^T \; \beta^T]$, are

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + S(\varepsilon)$$

$$\dot{\varepsilon} = -\begin{bmatrix} \eta^T \\ \alpha^T \\ \beta^T \end{bmatrix} \varepsilon + u_\varepsilon \quad , \quad \dot{\eta} = u_\eta \quad , \quad \dot{\alpha} = u_\alpha \quad , \quad \dot{\beta} = u_\beta \tag{5}$$

where u_ε, u_η, u_α, and u_β are random processes with the following a priori statistics

$$E\{u_\ell\} = 0 \quad , \quad E\{u_\ell(t)u_\ell^T(\tau)\} = q_\ell(t)\delta(t-\tau) \quad , \quad \ell\varepsilon\{\varepsilon,\eta,\alpha,\beta\} \quad .$$

(In the program, selection of this model corresponds to a value of the parameter, MODOP = 11.) The function, $S(\varepsilon)$, is defined in Table 1a.

Model 2. This model also employs a first-order Gauss-Markov process to approximate the thrust acceleration error components. The form is slightly different, employing fewer variables (and hence, has less flexibility), to yield the differential equations for the state vector, $X^T = [r^T \; v^T \; \varepsilon^T \; \alpha^T]$:

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + S(\varepsilon)$$

$$\dot{\varepsilon} = -\lceil \alpha \rfloor \varepsilon + u_\varepsilon \quad , \quad \dot{\alpha} = u_\alpha \tag{6}$$

where u_ε and u_α are random with

$$E\{u_\ell\} = 0 \quad , \quad E\{u_\ell(t) u_\ell^T(\tau)\} = q_\ell(t)\delta(t-\tau) \quad , \quad \ell \varepsilon \{\varepsilon, \alpha\} .$$

(This model corresponds to a value of MODOP = 21.) Again, $S(\varepsilon)$ is defined in Table 1a. The matrix $\lceil \alpha \rfloor$, is diagonal and its elements are those of the vector, α. This model is basically the same as Model 3, except that the appropriate state error covariance terms must be set to zero.

Model 3. The thrust acceleration error components are approximated by two first-order Gauss-Markov processes. The differential equations for the state vector, $X^T = [r^T \; v^T \; \varepsilon^T \; \eta^T \; \alpha^T \; \beta^T]$, are

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + S(\varepsilon, \eta)$$

$$\dot{\varepsilon} = -\lceil \alpha \rfloor \varepsilon + u_\varepsilon \quad , \quad \dot{\alpha} = u_\alpha$$

$$\dot{\eta} = -\lceil \beta \rfloor \eta + u_\eta \quad , \quad \dot{\beta} = u_\beta \tag{7}$$

where u_ε, u_η, u_α, u_β are purely random with

$$E\{u_\ell\} = 0 \quad , \quad E\{u_\ell(t)u_\ell^T(\tau)\} = q_\ell \delta(t-\tau) \quad , \quad \ell \epsilon \{\varepsilon,\eta,\alpha,\beta\} \quad .$$

(This model corresponds to MODOP = 21.) The vector function, $S(\varepsilon,\eta)$, is defined in Table 1a. The matrices, $\ulcorner \alpha \lrcorner$ and $\ulcorner \beta \lrcorner$, are diagonal matrices whose diagonal elements form the components of the vectors, α and β.

Model 4. The thrust acceleration error components are approximated by a second-order Gauss-Markov process. The differential equations for the state vector, $X^T = [r^T \; v^T \; \varepsilon^T \; \eta^T \; \alpha^T \; \beta^T]$, are

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + S(\eta)$$
$$\dot{\varepsilon} = -\ulcorner \alpha \lrcorner \, \varepsilon - \ulcorner \beta \lrcorner \, \eta + u_\varepsilon \tag{8}$$
$$\dot{\eta} = \varepsilon \quad , \quad \dot{\alpha} = u_\alpha \quad , \quad \dot{\beta} = u_\beta$$

where u_ε, u_α, and u_β are purely random processes which satisfy the *a priori* statistics

$$E\{u_\ell\} = 0 \quad , \quad E\{u_\ell(t)u_\ell^T(\tau)\} = q_\ell(t)\delta(t-\tau) \quad , \quad \ell \epsilon \{\varepsilon,\alpha,\beta\} \quad .$$

(This model corresponds to MODOP = 12.) The vector, $S(\eta)$, is defined in Table 1a. $\ulcorner \alpha \lrcorner$ and $\ulcorner \beta \lrcorner$ are diagonal matrices whose diagonal elements form the components of α and β.

Model 5. The thrust acceleration error is approximated by a slightly different form for the two first-order Gauss-Markov process. The differential equations for the state vector $X^T = [r^T \; v^T \; \varepsilon^T \; \eta^T \; \alpha^T \; \beta^T]$ are

$$\dot{r} = v \quad , \quad \dot{v} = -\frac{\mu}{|r|^3} r + S(\varepsilon,\eta)$$
$$\dot{\varepsilon} = -\begin{bmatrix} \alpha_1^2 & 0 & 0 \\ 0 & \alpha_2^2 & 0 \\ 0 & 0 & \alpha_3^2 \end{bmatrix} \varepsilon + u_\varepsilon \quad , \quad \dot{\alpha} = u_\alpha \tag{9}$$
$$\dot{\eta} = -\begin{bmatrix} \beta_1^2 & 0 & 0 \\ 0 & \beta_2^2 & 0 \\ 0 & 0 & \beta_3^2 \end{bmatrix} \eta + u_\eta \quad ; \quad \dot{\beta} = u_\beta$$

where u_ε, u_η, u_α, u_β are purely random with

$$E\{u_\ell\} = 0 \quad , \quad E\{u_\ell(t)u_\ell^T(\tau)\} = q_\ell(t)\delta(t-\tau) \quad , \quad \ell\varepsilon\{\varepsilon,\eta,\alpha,\beta\} \quad .$$

(This model corresponds to MODOP = 22.) $S(\varepsilon)$ is defined in Table 1a. The elements of α_i and β_i are the elements of the vectors, α and β.

The Vector Function, S

In all models, the function, S, may be defined in one of three ways, depending upon the value of the program parameter, KSW. (See also Figure 1.) Table 1a is a cross-reference matrix showing the form of S for various values of MODOP and KSW. (KSW is also called KAX in the program.)

Observations

Observations for the orbit determination process consist of range, ρ; range-rage, $\dot{\rho}$; and three celestial angles, λ_i, defined below.

λ_1 = sun-vehicle-planet angle

λ_2 = star-vehicle-planet angle

λ_3 = sun-vehicle-star angle

The true and nominal state vectors are used to compute the true and nominal observation values. Further, the true observations are corrupted by adding white noise to the deterministically computed values. The random components are obtained by sampling from known Gaussian distributions. If each observation, specified generically as Ω, is made at discrete times, t_i, the observation-state relationship can be expressed as

$$\Omega_i = G_\Omega(X_i, t_i) + \nu_i \tag{10}$$

The observation error, ν_{Ω_i}, is assumed to have the a priori statistics $E\{\nu_{\Omega_i}\} = 0$, $E\{\nu_{\Omega_i}\ \nu_{\Omega_j}\} = R_{\Omega_i}\ \delta_{i_j}$ where δ_{i_j} is the Kronecker delta.

Earth Ephemeris

The Earth's orbit is assumed to be circular, an assumption not unreasonable in light of other model dynamics employed. The position and velocity of the Earth is determined from the following equations:

$$
\begin{aligned}
\psi &= \mathrm{mod}\ (\omega_E t\ ,\ 2\pi) \\
X_E &= C \cos \psi \\
Y_E &= C \sin \psi \\
V &= C\ \omega_E \\
\dot{X}_E &= -V \sin \psi \\
\dot{Y}_E &= V \cos \psi
\end{aligned}
\tag{11}
$$

where C is the mean Earth-Sun distance (see Figure 4.).

Tracking Station Motion

Figure 4 shows the coordinate frames and variables used in defining motion of (up to three) tracking stations, due to the Earth's rotation. The Earth is assumed to rotate at a constant rate; no precession or nutation is accounted for. From the figure, the tracking station position and velocity in heliocentric coordinates are seen to be:

$$
\begin{bmatrix} X_s \\ Y_s \\ Z_s \end{bmatrix}_i = \begin{bmatrix} X_s' \\ Y_s' \\ Z_s' \end{bmatrix} + \begin{bmatrix} X_E \\ Y_E \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \varepsilon & -\sin \varepsilon \\ 0 & \sin \varepsilon & \cos \varepsilon \end{bmatrix} \begin{bmatrix} D_i \cos (\phi+\phi_{oi}) \\ D_i \sin (\phi+\phi_{oi}) \\ z_{s_i} \end{bmatrix} + \begin{bmatrix} X_E \\ Y_E \\ 0 \end{bmatrix}
$$

$$
\begin{bmatrix} \dot{X}_s \\ \dot{Y}_s \\ \dot{Z}_s \end{bmatrix}_i = \begin{bmatrix} \dot{X}_E \\ \dot{Y}_E \\ 0 \end{bmatrix} + \begin{bmatrix} -D_i \sin \phi_i\ (\Omega \cos \varepsilon + \omega) + z_{s_i}\ \Omega \sin \varepsilon) \\ D_i \cos \phi_i\ (\Omega + \omega \cos \varepsilon) \\ D_i \cos \phi_i\ \omega \sin \varepsilon \end{bmatrix}
$$

where ω is the Earth spin rate, Ω is the orbital angular rate of Earth, and $\phi_i = \omega t + \phi_{oi}$, and where the subscript, i, represents the i^{th} station.

For the geographic rectangular station coordinates x_{s_i}, y_{s_i}, z_{s_i}, D_{s_i} is $\sqrt{x_{s_i}^2 + y_{s_i}^2}$ for the i^{th} station; Ψ_{D_i} is the right ascension of the i^{th} station at the initial time. The obliquity of the ecliptic is ε; in the program $\sin \varepsilon = .3979 \ldots$ and $\cos \varepsilon = .9174 \ldots$ are coded directly

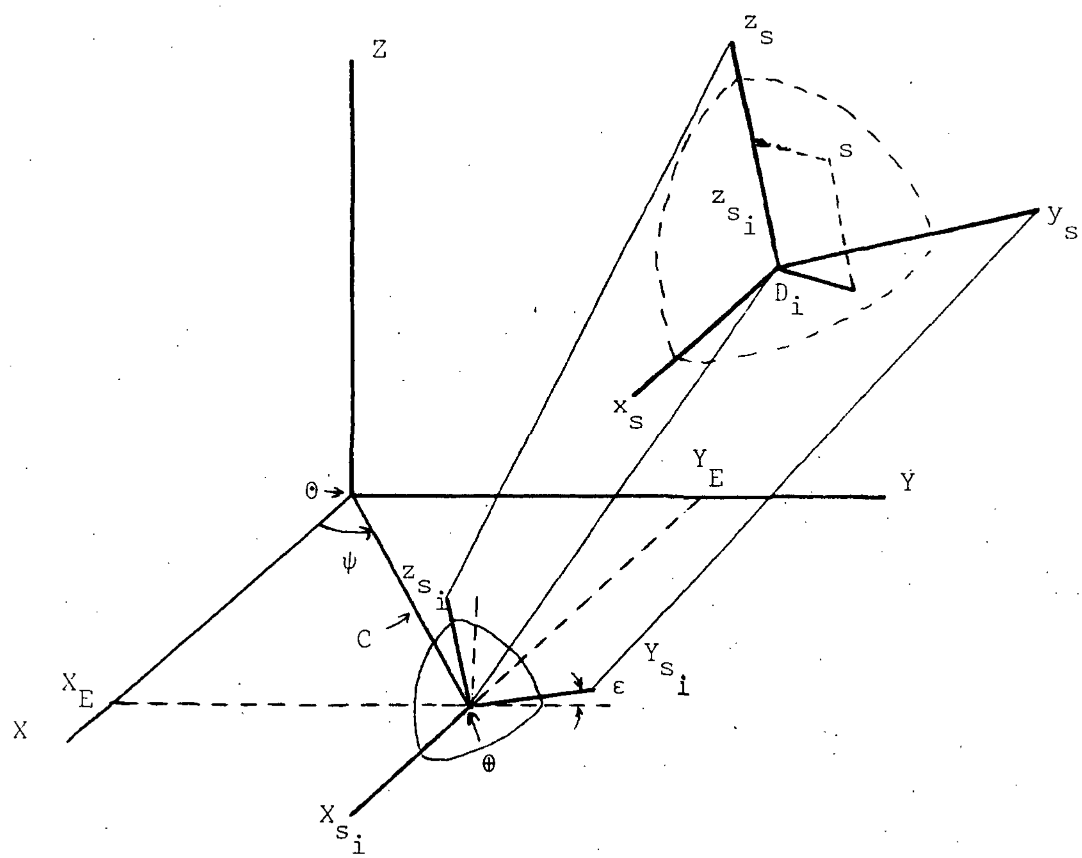

Figure 4. Earth Ephemeris and Tracking Station Geometry

Filter Equations

The differential equations described by any of the above models may be written in the following vector form to obtain the equations of state:

$$\dot{X} = F(X,t) \quad , \quad X(t_o) = X_o \tag{13}$$

The extended form of the Kalman-Bucy filter is used to obtain the state estimate. That is, given a previous estimate, X_{k-1}, and the associated state error covariance matrix, P_{k-1}, the estimate and error covariance at time, t_k, are obtained from the following equations:

$$\bar{X}_k = \hat{X}_{k-1} + \int_{t_{k-1}}^{t_k} F(\bar{X}(\tau), \tau)d\tau$$

$$\bar{P}_k = P_{k-1} + \int_{t_{k-1}}^{t_k} \dot{\bar{P}}(\tau)d\tau$$

$$\dot{\bar{P}} = A(t)\bar{P} + \bar{P}A^T(t) + Q(t) \tag{14}$$

$$K_k = \bar{P}_k H_k^T [H_k \bar{P}_k H_k^T + R_{\Omega k}]^{-1}$$

$$\hat{X}_k = \bar{X}_k + K_k[\Omega_k - G_\Omega(\bar{X}_k, t_k)]$$

$$P_k = [I - K_k H_k]\bar{P}_k \quad ,$$

where $A(t) = \partial F(\bar{x},t)/\partial x$ and $H_k = \partial G_\Omega(\bar{x}_k, t_k)/\partial x$, and $Q' \varepsilon \{q, q_\varepsilon, q_\eta, q_\alpha, q_\beta\}$, the appropriate non-zero submatrices of the state noise covariance, Q.

Program Description

Main Program - LOGO

The main program provides the controlling logic for driving the overall program. All problem input and batch mode control parameters are input here. Initialization of program options, and associated logic flags and parameter values are set. Plot option logic is set and the plot subroutine is called (on problem termination). Program results are saved on a separately specified output file for later use as inputs for continuation of problem at hand.

Language

FORTRAN IV

List of Subroutines used

FORTRAN IV	OUTPUT PATH UPDATE MOTION ACCEL AMATRIX HMATRIX OBSERV GAUSS	These are explained in this report
	MINIPLT	plotting routine for UT plotting system
COMPASS	RANF	UT2 system library; generates uniform random sequence
	MSAB	matrix multiplication $C = AB$
	MSABT	matrix multiplication $C = AB^T$
	MSATB	matrix multiplication $C = A^TB$
	IPRINT	available for error messages from the above 3 routines

Control Cards:

```
READPF <number> NAV  RMS

REWIND  NAV RMS

RUN  TX  I=NAV

COPYBF  RMS LGO

SETCORE  ZERO

LGO
```

Computing System

CDC 6600, UT-2 Operating System

Inputs

Input parameter values are entered under both formatted and NAMELIST input (READ) statements. They are explained in Table 2.

Outputs

All input parameters are printed immediately after being read. Upon completion of problem computations, selected variables are plotted (position, velocity, etc.) depending upon the plot option selected. Also, all input parameter values are saved on output file TAPE1, and can be subsequently used, if desired, as the input source for a later continuation run. This sequence occurs at the end of each run; in order to use TAPE1 as a new input file, it must be saved as a permanent file, or punched on cards.

Units

Any set of engineering units may be employed. All computation and output are done in the selected set with one exception. The choice of Earth radii and days as units of distance and time results in computation and print output in these units; however, plotting is in kilometers and seconds.

Input File Setup

Basic problem input data is entered via the NAMELIST INPUTS whose elements are listed in Table 2. Since the data is via NAMELIST, any unspecified parameters are automatically assigned values of zero if the SETCORE zero control card has been used. Figure 5a shows a sample data deck setup for the case where the a priori state error covariance matrix is diagonal. Figure 5b shows the setup for non-zero elements in the a priori error covariance. Note, in this case, the addition of NAMELIST APCOV, with elements made up of the array, P. Further, the array, STERCOV in NAMELIST INPUTS, must be zero in order to read in APCOV. (See Table 3.)

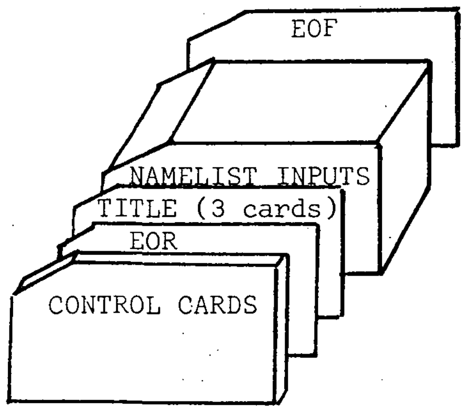

Figure 5a. Normal Deck Setup

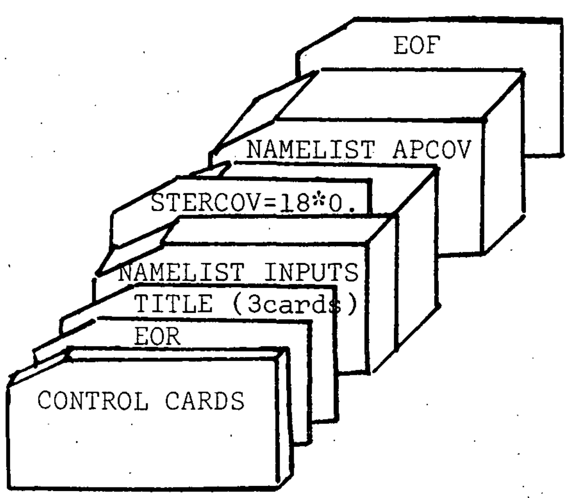

Figure 5b. Alternate Deck Setup

On-Line Feature

The on-line feature of the program is primarily that of presenting a running display on an assigned CRT terminal. Selection of the on-line feature is done by assigning a negative value to the parameter, PB, in NAMELIST INPUTS. Each time the output routine is entered, a colon, :, is displayed indicating readiness to receive a keyboard input. The keyboard input is one of the digits, 0 through 7; their meaning is shown in Table 3.

K/B Input	Definition
0	Return and continue problem computations
1	Print state deviation
2	Print state vector
3	Print thrust acceleration (local frame)
4	Print error covariance trace and RSS
5	Print observation error and H-matrix
6	Discontinue CRT display after next 0 entered
7	Revise problem termination time to the current time plus one integration step

Table 3. On-Line Feature Keyboard Inputs

Print Output

Printed output data consists of true (simulated) and nominal (estimated) state vector and state vector deviation, Earth state vector, error covariance state noise covariance, filter gain, observations, observation error, and the observation matrix, ∂ (observation)/∂ (state). A sample block of print is shown in Figure 6.

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
	STATE DEVIATION					
TRU	-4.717917E-04	2.706712E-04	-2.067224E-03	-1.668010E-03	7.685284E-04	-3.299164E-03
EST	-6.281400E-05	4.377924E-05	-6.082418E-04	-1.230145E-04	6.696892E-05	-6.641105E-04
ERR	-4.089777E-04	2.268919E-04	-1.458982E-03	-1.544996E-03	7.015595E-04	-2.635054E-03

STATE VECTORS

	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
TRU	2.2996036202E+04	5.7008273180E+03	1.2378635094E+01	-9.0470423135E+01	4.2370027592E+02	9.8827224301E-01
EST	2.2996036674E+04	5.7008270473E+03	1.2380702317E+01	-9.0468755125E+01	4.2369950740E+02	9.9157140721E-01

ϵ_1	ϵ_2	ϵ_3	θ_1	θ_2	θ_3
0.	1.9365773705E-02	0.	0.	0.	0.
α_1	α_2	α_3	β_1	β_2	β_3
0.	0.	0.	0.	0.	0.

Reference frame depend
on value of variable K
in subroutine ACCEL

EARTH	X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$
	2.284295E+04	5.322594E+03	0.	-9.162424E+01	3.932232E+02	0.

ACCELERATION ERRORS

	X	Y	Z	MAGNITUDE
ACCEL DEV, MAG	-5.7201338169E-05	2.3416733034E-02	8.8295810536E-05	2.3416969363E-02
PLAT MSALGN ERRS, MAG	0.	0.	0.	0.
ACCELRMTR ERRS, MAG	0.	0.	0.	0.
ESTMTD ACCELRNS, MAG	0.	1.9113461439E-02	0.	1.9113461439E-02

Local Frame

*No IMU code currently exists

THRUST ACCELERATION

	X	Y	Z
TRUE	-8.990747E-02	3.624317E-01	8.829581E-05
NOM/MEAS	-8.881649E-02	3.582686E-01	0.

AXT 3.7341674785E-01 AXN 3.6911346144E-01

TRUE MAGNITUDE NOMINAL MAGNITUDE

STATE ESTIMATE COVARIANCE

8E-08	-4E-08	4E-07	2E-07	-1E-07	5E-07	0	-2E-07	0	0	0	0	0	0	0	0	0	0
-4E-08	3E-08	-3E-07	-1E-07	6E-08	-4E-07	0	1E-07	0	0	0	0	0	0	0	0	0	0
4E-07	-3E-07	4E-06	8E-07	-4E-07	5E-06	0	-5E-07	0	0	0	0	0	0	0	0	0	0
2E-07	-1E-07	8E-07	1E-06	-5E-07	1E-06	0	-2E-06	0	0	0	0	0	0	0	0	0	0
-1E-07	6E-08	-4E-07	-5E-07	2E-07	-7E-07	0	7E-07	0	0	0	0	0	0	0	0	0	0
5E-07	-4E-07	5E-06	1E-06	-7E-07	1E-05	0	-8E-07	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
-2E-07	1E-07	-5E-07	-2E-06	7E-07	-8E-07	0	4E-06	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

SUBTRACES 4.4827E-06 1.0998E-05 4.0618E-06 0. 0. 0. RSS ERR 4.4207E-03

0	0	0	5E-06	0	5E-06	0	1E-05	0	0	0	0	0	0	0	0	0	0
X	Y	Z	$\dot{X}$	$\dot{Y}$	$\dot{Z}$	ϵ_1	ϵ_2	ϵ_3	θ_1	θ_2	θ_3	α_1	α_2	α_3	β_1	β_2	β_3

Diagonal elements of state noise covariance, Q

(If Q is non-diagonal, and adaptive estimation features are added, -IQ≠0, the full matrix is

GAIN 2.115505E+00 -1.474435E+00 2.048490E+01 4.142989E+00 -2.255438E+00 2.236650E+01 0. -3.041013E+00 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.

WL 2.031357E+09 HPHT 2.228169E-11 HHT 0.

OBSERVATION DATA

STAR-PLANT

	ρ				
TRUE	2.698708E+00	NOMINAL	2.698738E+00	ERROR	-2.969220E-05

	$\partial G/\partial X$	$\partial G/\partial Y$	$\partial G/\partial Z$	$\partial G/\partial \dot{X}$	$\partial G/\partial \dot{Y}$	$\partial G/\partial \dot{Z}$
HMATRIX	1.229124E-03	-5.643260E-04	2.041982E-03	0.	0.	0.

1 ITERATIONS ON STATE ESTIMATE

Figure 6. Sample Printer Output

Flow Diagram

Figure 7 is a functional flow diagram of the main program logic.

Variables

Tables 2a and 2b define the NAMELIST INPUTS and APCOV parameters. Tables 4 through 14 define the labeled common blocks (there is no blank COMMON). Table 15 defines those internal variables not contained in Tables 2 through 14. These internal variables are in alphabetical order.

Subroutine OUTPUT

This subroutine provides the logic for controlling print output as well as the on-line CRT display logic. Data to be output is communicated to the routine via labeled COMMON. Figure 8 is a functional flow diagram of the subroutine.

Subroutine PATH

Subroutine PATH performs the numerical integration of the equations of motion, and stores the data points for plotting. Two orders of integrators are available: Fourth-order Runge-Kutta-Gill, and second-order Runge-Kutta. The type of integrator is specified by the "hard-coded" value of the parameter, IGO.

IGO = 2, Second-Order Runge-Kutta

$$\bar{y}_{i+1} = y_i + f(y_i, t_i)h$$

$$y_{i+1} = y_i + [f(\bar{y}_{i+1}, t_{i+1}) - f(y_i, t_i)]\frac{h}{2}$$

IGO = 4, Fourth-Order Runge-Kutta-Gill

$$c_1 = 1 - \sqrt{2}/5 \qquad c_5 = -(2 + 3/\sqrt{2})$$

$$c_2 = -2 + 3\sqrt{2} \qquad c_6 = 2 + \sqrt{2}$$

$$c_3 = 2 - \sqrt{2} \qquad c_7 = 1/6$$

$$c_4 = 1 + 1/\sqrt{2} \qquad c_8 = -1/3$$

$$1\begin{cases} g_1 = f(y_i, t_i) \\ \bar{y} = y_i + g_1 \dfrac{h}{2} \end{cases}$$

$$2\begin{cases} g_2 = f(\bar{y}, t_{i+\frac{1}{2}}) \\ \bar{y} = \bar{y} + c_1(g_2 - g_1)h \\ g_1 = c_2 g_1 + c_3 g_2 \end{cases}$$

$$3\begin{cases} g_2 = f(\bar{y}, t_{i+\frac{1}{2}}) \\ \bar{y} = \bar{y} + c_4(g_2 - g_1)h \\ g_1 = c_5 g_1 + c_6 g_2 \end{cases}$$

$$4\begin{cases} g_2 = f(\bar{y}, t_{i+1}) \\ y_{i+1} = \bar{y} + (c_7 g_2 + c_8 g_1)h \end{cases}$$

Figure 9 is a flow diagram of the numerical integration logic flow.

Subroutine UPDATE

Subroutine UPDATE computes the Earth ephemeris data and tracking station motion, provides the logic controlling computation of the observations and observation partial derivatives, and computes the filtered state estimate. Figure 10 shows a general logic flow for this routine.

Subroutine MOTION

In addition to calling subroutine ACCEL, which computes the dynamic accelerations, the model compensation differential equations are evaluated. Further, the error covariance matrix differential equation is evaluated. Figure 11 is a flow diagram of the corresponding logic. See also Table 1.

Subroutine ACCEL

The true (simulated) and nominal accelerations (gravitational and thrusting) are computed according as the parameter, JP, equals 1 or 2, respectively. This includes both the gravitational and the thrust accelerations. For the nominal acceleration, the coordinate system used depends upon the value of the parameter, KSW.

KSW = 1 , Heliocentric rectangular coordinates

2 , Local rectangular coordinates

3 , Local angular coordinates

Figure 12 is a flow chart of the subroutine.

Subroutine AMATRIX

The partial derivatives of the state differential equations are computed. The forms of the derivatives depend on the coordinate system (KSW = 1, 2, or 3), and on the type of equations used to model the nominal thrust acceleration errors (MODOP = 11, 12, 21, 22). Figure 13 is a flow diagram for the subroutine.

Subroutine HMATRIX

The partial derivatives of the observations are computed and provided to subroutine UPDATE for use in the filter equations. Figure 14 is the flow diagram.

Subroutine OBSERV

The observations (range, range-rate, sun-planet angle, star-planet angle, and/or sun-star angle) are computed. Both the true and nominal observation values are determined. Figure 15 is the flow diagram.

Subroutine GAUSS

This routine provides random samples from a standardized normal (Gaussian) distribution.

Subroutine MINIPLT

(See Department of Aerospace Engineering library documentation.)

KSW

S	1	2	3
11	$T^* + \varepsilon$	$T^* + R\varepsilon$	$T^* + R[a^*+\varepsilon_1]\begin{bmatrix}\sin\varepsilon_2\cos\varepsilon_3\\ \cos\varepsilon_2\\ \sin\varepsilon_2\sin\varepsilon_3\end{bmatrix}$
21	$T^* + \varepsilon + \eta$	$T^* + R[\varepsilon + \eta]$	$T^* + R[a^*+\varepsilon_1+\eta_1]\begin{bmatrix}\sin(\varepsilon_2+\eta_2)\cos(\varepsilon_3+\eta_3)\\ \cos(\varepsilon_2+\eta_2)\\ \sin(\varepsilon_2+\eta_2)\sin(\varepsilon_3+\eta_e)\end{bmatrix}$
12	$T^* + \eta$	$T^* + R\eta$	$T^* + R[a^*+\varepsilon_1]\begin{bmatrix}\sin\varepsilon_2\cos\varepsilon_3\\ \cos\varepsilon_2\\ \sin\varepsilon_2\sin\varepsilon_3\end{bmatrix}$
22	$T^* + \varepsilon + \eta$	$T^* + R[\varepsilon + \eta]$	$T^* + R[a^*+\varepsilon_1+\eta_1]\begin{bmatrix}\sin(\varepsilon_2+\eta_2)\cos(\varepsilon_3+\eta_3)\\ \cos(\varepsilon_2+\eta_2)\\ \sin(\varepsilon_2+\eta_2)\sin(\varepsilon_3+\eta_3)\end{bmatrix}$

Table 1a. Selection Matrix for Vector Function, S

<table>
<tr><th>MODOF</th><th>MODEL</th><th>with IQ = 0</th><th>Noise Mapping Matrix</th></tr>
<tr><td>0</td><td>0</td><td>$\dot{\bar{P}} = A\bar{P} + \bar{P}A^T + \Gamma Q \Gamma^T$</td><td rowspan="6">$\Gamma = \begin{bmatrix} I & & & & & \\ & R & & & & \\ & & I & & & \\ & & & I & & \\ & & & & I & \\ & & & & & I \end{bmatrix}$

$R = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$</td></tr>
<tr><td>1</td><td>0</td><td>$\dot{\bar{P}} = A\bar{P} + \bar{P}A^T + Q$</td></tr>
<tr><td>11</td><td>1</td><td rowspan="4">$\dot{\bar{P}} = A\bar{P} + \bar{P}A^T + \Gamma Q \Gamma^T$</td></tr>
<tr><td>12</td><td>4</td></tr>
<tr><td>21</td><td>2 or 3</td></tr>
<tr><td>22</td><td>5</td></tr>
</table>

Table 1b. State Error and State Noise Covariance Logic

Code	Dim	Type	Text	Description
WEARTH	1	R	ω_E	Orbital angular velocity of the Earth
AU	1	R	C	1 Astronomical Unit (a.u.)
PROBE	18	R	X	Initial value of state vector to be estimated
				(1) - (6): Position and Velocity, $X\ Y\ Z\ \dot{X}\ \dot{Y}\ \dot{Z}$
				(7) - (24): $\varepsilon_1\ \varepsilon_2\ \varepsilon_3\ \eta_1\ \eta_2\ \eta_3\ \alpha_1\ \alpha_2\ \alpha_3\ \beta_1\ \beta_2\ \beta_3$
PROBER	6	R		Initial state vector errors in position and velocity
GSUN	1	R	μ	Gravitational parameter of the sun
TSTART	1	R		Initial time
TSTOP	1	R		Final (problem termination) time
TRANGE	1	R		Range observation interval
TRNGRT	1	R		Range rate observation interval
TSNPLT	1	R		Sun-planet angle observation interval
TSTRPLT	1	R		Star-planet angle observation interval
TSNSTR	1	R		Sun-star angle observation interval
THRSTAX	1	R	a*	Nominal value of thrust acceleration
THRSTER	4	R		(1): Frequency (cycles/sec) of sinusoidal acceleration error variation
				(2): Percent of nominal thrust acceleration magnitude to be used as magnitude error
				(3): Standard deviation for random component of thrust acceleration error magnitude
				(4): Standard deviation (radians) for random component of thrust acceleration pointing error
ACCELER	6	R		Not used
PLATFER	6	R		Not used
STERCOV	18	R	P	Initial values of diagonal elements of error covariance
STNSCOV	18	R	Q	Constant values of diagonal elements of state noise covariance
MODOP	1	I		Model option selection parameter

Table 2a. NAMELIST/INPUTS/

Code	Dim	Type	Text	Description
				MODOP = 0, 1 - Model 0 MODOP = 21 - Model 3 MODOP = 11 - Model 1 MODOP = 12 - Model 4
MSIMOP	1	I		Not used
OBSCOV	5	R	R_{Ω_i}	Nominal observation error covariance (elements 1-5 correspond to the elements for TRANGE, TRNGRT, TSNPLT, TSTRPLT, TSNSTR, respectively)
TOBSCOV	5	R		True observation error covariance (elements correspond to those above (OBSCOV))
STAR	3	R		Unit vector pointing to navigation star
DTINT	1	R	h	Integration step size
IPRNT	5	R		Print frequency for each observation type
UNMOD	12	R		Initial values of model compensation parameters
IPLOTYP	1	I		Plot type flag: Values may be any number of digits from 1 to 7, in any order. Thus, 127 or 271, etc. results in plots on printer paper, teletype, or CRT, and microfilm. Codes are: 1 = line printer; 2 = teletype; 3 = 12" paper, ball point pen; 4 = 12" paper ink pen; 5 = 30" paper ball point pen; 6 = 30" paper ink pen; 7 = 35 mm film.
TOBS		R		Initial times for each observation type
TUP	1	R		Flag for controlling initial observation processing TUP < 0. - Process first observation(s) at initial time prior to integrating first step TUP ≥ 0. - Integrate first step then process observations
MAXIT	1	I		Not used
E	6	R		E(1) contains age-weighting parameter, $1 \leq S < \infty$, if desired. E(2)-E(6) not used.
KAX	1	I		Flag controlling coordinate system for compensating models: KSW = 1 Heliocentric rectangular coordinate (X Y Z) KSW = 2 Local rectangular coordinates, (x y z) KSW = 3 Local angular coordinates (ϕ γ θ)

Table 2a (Cont'd.)

Code	Dim	Type	Text	Description
IQ				Not used
KAGE	1	I		Flag controlling age-weighting suboptimal filter KAGE < 0, use suboptimal age-weighting (not used)
PB	1	R		On-line control parameter PB < 0, employ on-line display
JPLOT	1	I		Not used
AUX	6	R		Auxiliary acceleration error simulation parameters (1) = s (rate of motion of tip of accel. vector) (2) = $\bar{\gamma}$ (max value of γ specifying boundary) (3) = $\bar{\theta}$ (initial value of θ; updated at each $\bar{\gamma}$ contact) (4) = ϕ (initial value of ϕ, direction of accel. (5) = σ_θ (standard deviation of the noise in θ) (6) = σ_s (standard deviation of the noise in s)
BETA	5	R		Not used
DZT	18	R		Not used
ZS	3	R	zs	Z-component (geographic rectangular coordinates) of tracking station
D	3	R	D_i	$\sqrt{x_{si}^2+y_{si}^2}$
W	4	R	w_E, ϕ_{oi}	w(1) = Rotational angular velocity of Earth; w(2)-w(4) = Initial angular displacements of tracking stations

Table 2a. (Cont'd.)

Code	Dim	Type	Text	Description
P	18,18	R	P	Complete 18 x 18 a priori state error covariance matrix (3 2 4 elements). If this NAMELIST is used NAMELIST/INPUTS/ must precede /APCOV/; the array STERCOV must be zero, otherwise /APCOV/ will not be read

Table 2b. NAMELIST/APCOV/

Code	Dim	Type	Text	References	Description
DV	200	R		LOGO, OUTPUT, PATH, UPDATE, MOTION, ACCEL,	State vectors and error covariance/ (1)-(6): True position and velocity (7)-(24): Nominal position and velocity, and compensating model parameters (25)-(195): Upper triangular elements of error covariance matrix: DV(K) = P(II,JJ) where I = MINO(II,JJ), J = MAXO(II,JJ), and K = (18-I)*(I-1) + (I*(I+1))/2 + J - I + 24
ZT	6	R			True position and velocity
Z	18	R	X		Nominal (estimated) state vector
DZ	18	R			Estimated state vector deviation
TSTART	1	R			Initial time
TEND	1	R			Problem termination time
T	1	R			Problem current time
WEARTH	1	R			Angular orbit velocity of Earth
AU	1	R	C		One astronomical unit (a. u.)
GSUN	1	R	μ		Gravitational parameter of the sun
MODOP	1	I			Model selection option parameter MODOP = 0, 1 - Model 0 MODOP = 21 - Model 3 MODOP = 11 - Model 1 MODOP = 12 - Model 4 MODOP = 21 - Model 2 MODOP = 22 - Model 5
MSIMOP	1	I			Not used
DTOBS	5	R			Observation intervals (Also, see TRUOBS, internal variables)

Table 4. COMMON/PARAMS/

Code	Dim	Type	Text	References	Description
KK	5	I			Print frequency control
OBSCOV	5	R	R_{Ω_i}		Nominal observation error covariance
TOBSCOV	5	R	R_{Ω_i}		True observation error covariance
STAR	3	R			Unit vector in direction of navigation star
ACE	12	R			Not used
THRSTAX	1	R	a*		Nominal thrust acceleration value
THRSTER	4	R			Thrust acceleration error simulation variables
P	18,18	R	P		State error covariance
STNSCOV	18	R	Q		State noise covariance (diagonal)
XE	6	R			State vector (position and velocity) of Earth
OBS	5	R	Ω_i		True observations
CALOBS	5	R	$G(\chi)$		Nominal (calculated)oobservations
GT	6	R			True and nominal thrust accelerations
DTINT	1	R	h		Integrator step size
HH	5,6	R	H		Observation matrix, $\partial G/\partial \chi$
A	12,18	R	A		State partial derivatives matrix, $\partial F/\partial \chi$
TOBS	5	R			Initial observation times

Table 4. (Cont'd.)

Code	Dim	Type	Text	References	Description
Y	703,6	12		LOGO, OUTPUT, PATH	Dependent parameters for plotting: (1-350,1): Position error norm (351-700,1): Position error variance $\sqrt{Tr}$ (1-350,2): Velocity error norm (351-700,2): Velocity error variance $\sqrt{Tr}$ (1-350,1) - (1-350,3): True thrust accel. errors (351-700,1) - (351-700,3): Nom. thrust accel. errors
X	703	R			(1-350), (351-700): T - TSTART (mission time)
MM	1	I			Number of points for plotting
L	1	I			Plot increment or frequency

Table 5. COMMON/PLOTS/

Code	Dim	Type	Text	References	Description
WLE	18	R		LOGO, OUTPUT	Not used
E	6	R			E(1) is the suboptimal filter parameter for age-weighting; E(2)-E(6) not used
STERCOV	18	R			Initial values of diagonal elements of error covariance
KSOP	1	I			Flag for determining suboptimal or optimal filter mode (≤ 0, optimal filter)

Table 6. COMMON/SUBOPT/

Code	Dim	Type	Text	References	Description
IQ	1	I		LOGO, OUTPUT	Flag for adaptive (Q) estimation. Not Used
Q	18,18	R	Q	UPDATE, MOTION	State noise covariance matrix (square); used only for diagonal terms. IQ should be set to zero.

Table 7. COMMON/STEST/

Code	Dim	Type	Text	References	Description
TUP	1	R		LOGO, PATH, UPDATE	Flag for determining initial observation logic
MAXIT	1	I			Not used
DZI	18	R			Not used

Table 8. COMMON/UPIT/

Code	Dim	Type	Text	References	Description
AUX	6	R		LOGO, UPDATE ACCEL	Parameters used in thrust acceleration error simulation
ZS	3	R	z_{s_i}		Z-axis coordinate of station location(s)
D	3	R	D_i		$\sqrt{x_{s_i}^2+y_{s_i}^2}$
W	4	R	ω_E,ϕ_{o_i}		W(1) = Angular rotational velocity of Earth; W(2)-W(4) = Initial angular displacements of stations

Table 9. COMMON/AUXL/

Code	Dim	Type	Text	References	Description
KSW, KAX	1	I		LOGO, UPDATE, MOTION, ACCEL, AMATRIX	Flag determining coordinate system for thrust acceleration error estimation
TX	4	R			(See CX1, CX3, SX2, SX3 in Internal Variables)

Table 10. COMMON/AXMOD/

Code	Dim	Type	Text	References	Description
ETA	6	R		OUTPUT, PATH, MOTION, ACCEL	(1)-(3): True thrust acceleration errors (4)-(6): Estimated thrust acceleration errors

Table 11. COMMON/TAE/

Code	Dim	Type	Text	References	Description
WK	18	R	K	OUTPUT, UPDATE	Kalman gain
WL	1	R			Working parameter = $(HPH^T + R)^{-1}$
HPHT	1	R	HPH^T		Working parameter
HHT	1	R	HH^T		Working parameter
TZ	18,18	R			Working array, see T, Internal Variables

Table 12. COMMON/FILT/

Code	Dim	Type	Text	References	Description
ISW				PATH, MOTION,	Not used
IGO				ACCEL	Flag specifying numerical integrator (2nd order or 4th order)

Table 13. COMMON/SW/

Code	Dim	Type	Text	References	Description
GRNDM	9	R		PATH, MOTION	Array of normally distributed random numbers
URNDM	9	R		ACCEL	Array of uniformly distributed random numbers

Table 14. COMMON/RNDM/

Code	Dim	Type	Text	References	Description
A	3	R		OBSERV	Range Vector
AB	1	R		OBSERV	Dot product of range vector and negative vehicle position vector
ADEV	1	R	δa	OUTPUT	Magnitude of true acceleration errors
AT	3	R		ACCEL	True thrust acceleration vector
AXT	1	R	a	OUTPUT	True thrust acceleration magnitude
AXN	1	R	a^*	OUTPUT	Nominal thrust acceleration magnitude
AO	1	R	a	ACCEL	True thrust acceleration magnitude

Table 15. Internal Variables

Code	Dim	Type	Text	References	Description
B	1	R		AMATRIX	Working parameter: $-\mu/\|r\|^3$
B	3	R		OBSERV	Negative of vehicle position vector
C	1	R		AMATRIX	Working parameter: $3\mu/\|r\|^5$
C	8	R	c_i	PATH	Coefficients in numerical integration equations
CA	1	R		HMATRIX	Cosine of sun-planet angle
CB	1	R		HMATRIX	Cosine of star-planet or star-sun angles
CE	1	R		MOTION	Dummy argument for function statement
CE	1	R	COS ε	UPDATE	Cosine of obliquity of the ecliptic
CG	1	R	COS γ	ACCEL	Cosine of angle between true acceleration vector and local (vehicle) y-axis
CL	1	R		OUTPUT	Hollerith character (colon, :) for on-line display
COB	1	R		UPDATE	Cosine of angle between line of sight to spacecraft and horizon
CP	1	R	COS ϕ	ACCEL	Cosine of angle, ϕ, used in simulating true thrust acceleration
CPHI	1	R	COS ψ	UPDATE	Cosine of heliocentric orientation angle
CPHI	1	R	COS$(\phi_i+\phi_{oi})$	UPDATE	Cosine of $\omega_E t + \phi_{oi}$ (for station position)
CXZ	1	R	COS$(\varepsilon_2$ or $\eta_2)$	ACCEL	Cosine of approximation to γ for KSW = 3
CX3	1	R	COS$(\varepsilon_3$ or $\eta_3)$	ACCEL	Cosine of approximation to θ for KSW = 3
CXN	3	R		ACCEL	Working array for AMATRIX operations
D	1	R	d	ACCEL	Distance of tip of acceleration vector (simulated) from last boundary contact

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
DCP	1	R	$D_i\cos(\phi_i+\phi_{oi})$	UPDATE	Distance of station from Earth's axis multiplied by cosine of $\omega_E t + \phi_{oi}$
DELT	1	R		PATH	Observation interval
DPLT	1	R		LOGO	Minimum observation interval
DSP	1	R	$D_i\sin(\phi_i+\phi_{oi})$	UPDATE	Station distance from Earth's axis times $\cos(\phi_i+\phi_{oi})$
DSV	1	R		UPDATE	Saved value of last observation interval
DX	6	R		OBSERV	Range and range rate vectors
DXE	6	R		HMATRIX	Range and range rate vectors
DXI	6	R		HMATRIX	Difference between vehicle position vector and Earth position vector; or vehicle position vector according as observation is star-planet/ sun-planet, or sun-star-angle respectively
DY	6	R		UPDATE	Observation deviation
E	1	R		OBSERV	Dot product of unit vector ot navigation star and range vector
EPS	1	R		PATH	Tolerance
EST	1	R		OUTPUT	Magnitude of estimated acceleration error
F1, F2	200	R		PATH	Working arrays for numerical integration
GAMMA	1	R	γ	ACCEL	Angle between simulated thrust acceleration vector and local (vehicle) y-axis
H	1	R	h	PATH	Step size for numerical integrations
H(6)	1	R		UPDATE	Row of H-matrix, and working array
HAF	1	R	h/2	PATH	Half of step size

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
I	1	I		All but GAUSS	Do-loop and array indices
II	1	I		UPDATE	Do-loop index
IFLAG	1	I		GAUSS	Logic flag
IGO	1	I		OUTPUT	Input (K/B) test parameter for on-line feature
INFO	24	A		LOGO	Problem/run title/identification
IPLTSV, IPL1	1	I		LOGO	Working parameters for plotting
IPRNT	1	I		PATH	Print control parameter in integration routine
IX	1	I		MOTION	Dummy index for function statement
J	1	I		LOGO, MOTION AMATRIX	Do-loop and array indices
JALL	1	I		OUTPUT, UPDATE	Parameter for controlling print information
JJ	1	I		UPDATE, MOTION	Do-loop index and dummy function argument
JP	1	I		MOTION	Flag determining computation of true or nominal thrust acceleration
JX	1	I		MOTION	Dummy argument for function statement
J1, J2	1	I		OUTPUT	Array indices
K	1	I		OUTPUT, UPDATE MOTION, OBSERV	OUTPUT argument, array index, function statement, OBSERV argument
K	7	A		LOGO	Array for plot identification parameters
KK	1	I		MOTION	Array index
KCHECK	1	I		HMATRIX	Parameter which flags singularity

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
KETA	1	I		MOTION	Array index
KL	1	I		MOTION	Counter
KM	1	I		LOGO	Number of graphs to be generated
KNT	1	I		PATH	Print frequency counter
KOP	1	I		HMATRIX	HMATRIX argument identifying observation type
K1, K2	1	I		ACCEL	Array indices identifying estimated accelerations
L	1	I		UPDATE	Array index
LPRNT	5	I		UPDATE	Print frequency counters
M	1	I		UPDATE, LOGO	Array indices
M1, M2	1	I		PATH	Array indices
N	1	I		LOGO	Array index
NTS	1	I		UPDATE	Number of tracking stations
N1	1	I		UPDATE, PATH	Array index; lower limit of do-loop index
N2	1	I		PATH	Upper limit of do-loop index
N6, N18	1	I		UPDATE	Array indices
OBSER	1	R		OUTPUT	Observation error
OBSTYP	5	R		OUTPUT	Observation type
OUT 1	6	R		OUTPUT	True state deviation
OUT 2	6	R		OUTPUT	Error in state deviation estimate
P	1	R		GAUSS	Parameter for sampling normal distribution

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
P(6)	1	R		ACCEL	Vector of true or nominal accelerations
P	200	R		MOTION	Array containing all the derivatives. P(1) -P(6): X Y Z $\dot{X}$ $\dot{Y}$ $\dot{Z}$ for simulated trajectory P(7) -P(24): X Y Z $\dot{X}$ $\dot{Y}$ $\dot{Z}$ $\dot{\varepsilon}_1$ $\dot{\varepsilon}_2$ $\dot{\varepsilon}_3$ $\dot{\eta}_1$ $\dot{\eta}_2$ $\dot{\eta}_3$ $\dot{\alpha}_1$ $\dot{\alpha}_2$ $\dot{\alpha}_3$ $\dot{\beta}_1$ $\dot{\beta}_2$ $\dot{\beta}_3$ for nominal state vector P(25)-P(195): Contains upper triangular portion of error covariance derivative. Statement functions are used to locate an element of the error covariance derivative as follows: PX(II,JJ) = DV(K(MINO(II,JJ),MAXO(II,JJ)) + 24) PX(II,JJ) looks like an array, and essentially corresponds to the elements of the error covariance matrix. DV is an array explained under COMMON/PARAMS/. MINO and MAXO are FORTRAN library functions. The statement function, K, matches an element p_{ij}, of the full error covariance matrix, to the appropriate position in the upper triangular matrix as it would be stored in a singly dimensioned array. K(IX,JX) = (18-IX)*(IX-1) + (IX*(IX-1))/2 + JX - IX Thus, $p_{59} = p_{95}$ (due to symmetry) and occupies location 66 (always choose IX ≤ JX as indicated in the PX function above).
PP	18,18	R		UPDATE	Working array for use in filter equations
PHI	1	R	ψ	UPDATE	Heliocentric orientation angle
PI	1	R		MOTION	Working parameter, $1/\sqrt{X^2+Y^2}$
PI2	1	R		ACCEL	$\pi/2$
PN	3	R		MOTION	Nominal acceleration values

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
PSIT	1	R		MOTION	Angle, computed from mission start, representing thrust acceleration, time correlation error: PSIT = PSI (f) = AMOD ($2\pi \cdot f \cdot \Delta t$, 2π)
PT	6	R		MOTION	True (simulated) acceleration values
PTEST	1	R		LOGO	Parameter used for testing for zero covariance
PX2, PY2	1	R		AMATRIX	Working parameters
R	1	R		ACCEL, AMATRIX OBSERV, GAUSS	Magnitude of position vector; in GAUSS R is a sample from a normalized uniform distribution
RR	1	R		ACCEL	Working parameter: $1/\lvert r\rvert$, $1/\lvert r\rvert^3$, etc.
RHO	1	R		HMATRIX, OBSERV	Range
RHOSQ	1	R		HMATRIX	Square of range
RMAG	1	R		UPDATE	Magnitude of vector from geocenter to station
RN	1	R		UPDATE	Sample of uniform distribution, 0 to 1
RORHO	1	R		HMATRIX	Magnitude of r over range
RP, RP2	1	R		AMATRIX	Working parameters
RRHO	1	R		HMATRIX	Magnitude of r times range
RSQ	1	R		HMATRIX	Square of magnitude of r
RSS	1	R		OUTPUT	Root sum square of covariance matrix
R2, R3, R5	1	R		AMATRIX	Working parameters: $1/\lvert r\rvert^2$, $1/\lvert r\rvert^3$, $1/\lvert r\rvert^5$
S	1	R		UPDATE	Age-weighting parameter for suboptimal filtering
S	3	R		OBSERV	Unit vector toward navigation star

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
SA	1	R		HMATRIX	Sin of sun-planet angle
SB	1	R		HMATRIX	Sin of star-planet or star-sun angles
SE	1	R		UPDATE	Sin of obliquity of ecliptic
SGCT	1	R	$\sin\gamma\cos\theta$	ACCEL	Trigonometric function of thrust pointing angles
SGST	1	R	$\sin\gamma\cos\theta$	ACCEL	Trigonometric function of thrust pointing angles
SIGA	1	R		LOGO	1σ value for thrust acceleration error magnitude random component
SMAG	1	R		UPDATE	Magnitude of range vector
SP	1	R	$\sin\phi$	ACCEL	Sine of angle ϕ for simulating true thrust acceleration error
SPHI	1	R	$\sin\psi$	UPDATE	Sine of heliocentric position angle
SQRTOBS	1	R		UPDATE	Square root of true observation error variance
SX2	1	R	$\sin(\varepsilon_1$ or $\eta_2)$	ACCEL	Sine of approximation to γ for KSW = 3
SX3	1	R	$\sin(\varepsilon_3$ or $\eta_3)$	ACCEL	Sine of approximation to θ for KSW = 3
T	1	R		GAUSS	Parameter used in obtaining random Gaussian sample
T	18,18	R		MOTION	Working array containing AP (for forming $\dot{P}$)
TA	1	R		ACCEL	Estimation thrust acceleration $a^* + (\varepsilon_1$ and/or $\eta_1)$
TAT1, TAT2, TAT3	1	R		MOTION	Working parameters
TDIF	1	R		PATH	Test parameter equal to final time - current time
THETA	1	R	θ	ACCEL	Angle used in orienting thrust vector

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
TM	1	R		MOTION	Current time - start time
TR	6	R		OUTPUT	Subtraces (3 elements each) of 18 x 18 covariance
TRUOBS	5	R	Ω_i	UPDATE	True (simulated) observations (1)=Range, (2)=Range-rate, (3)=Sun-planet, (4)= star-planet angle, (5)=Sun-star angle
TSAVE	1	R		UPDATE	Working parameter used in determining obs. time
TSV	1	R		ACCEL	Working parameter: time thrust vector last encountered its boundary
TSTOP	1	R		PATH	Next observation time ($=t_{current}$ + observ. interval)
TWIPI	1	R		ACCEL	2π
TWOPI	1	R		MOTION,UPDATE	2π
T2	1	R		GAUSS	Working parameter
V	1	R		UPDATE	Earth's orbital speed
WCE	1	R		UPDATE	Earth's angular orbital velocity times cos ε
WSE	1	R		UPDATE	Earth's angular orbital velocity times sin ε
X	1	R		GAUSS	Random sample from Gaussian distribution
XDUM	6	R		UPDATE	Dummy array containing all zeros
XE	6	R		HMATRIX	Heliocentric station location and velocity
XI	6	R		OBSERV	Earth state vector or zero vector depending upon whether or not celestial obs. involves Earth
XIN	6	R		HMATRIX	Earth state vector or zero vector depending upon whether or not celestial obs. involves Earth
XN	18	R		AMATRIX	Nominal (estimated) state vector

Table 15. (Cont'd.)

Code	Dim	Type	Text	References	Description
XNOISE	1	R		UPDATE	Random sample from Gaussian distribution
XS	6	R		UPDATE, OBSERV	Heliocentric station location and velocity
XT	6	R		ACCEL	State vector (position and velocity)
XV	6	R		OBSERV	Vehicle state vector (position and velocity)
XY	1	R		AMATRIX	Working parameter
Y	5	R	Ω_i	OBSERV	Observations
ZETA	1	R		ACCEL	Working parameter (angle) used in determining simulated thrust acceleration

Table 15. (Cont'd.)

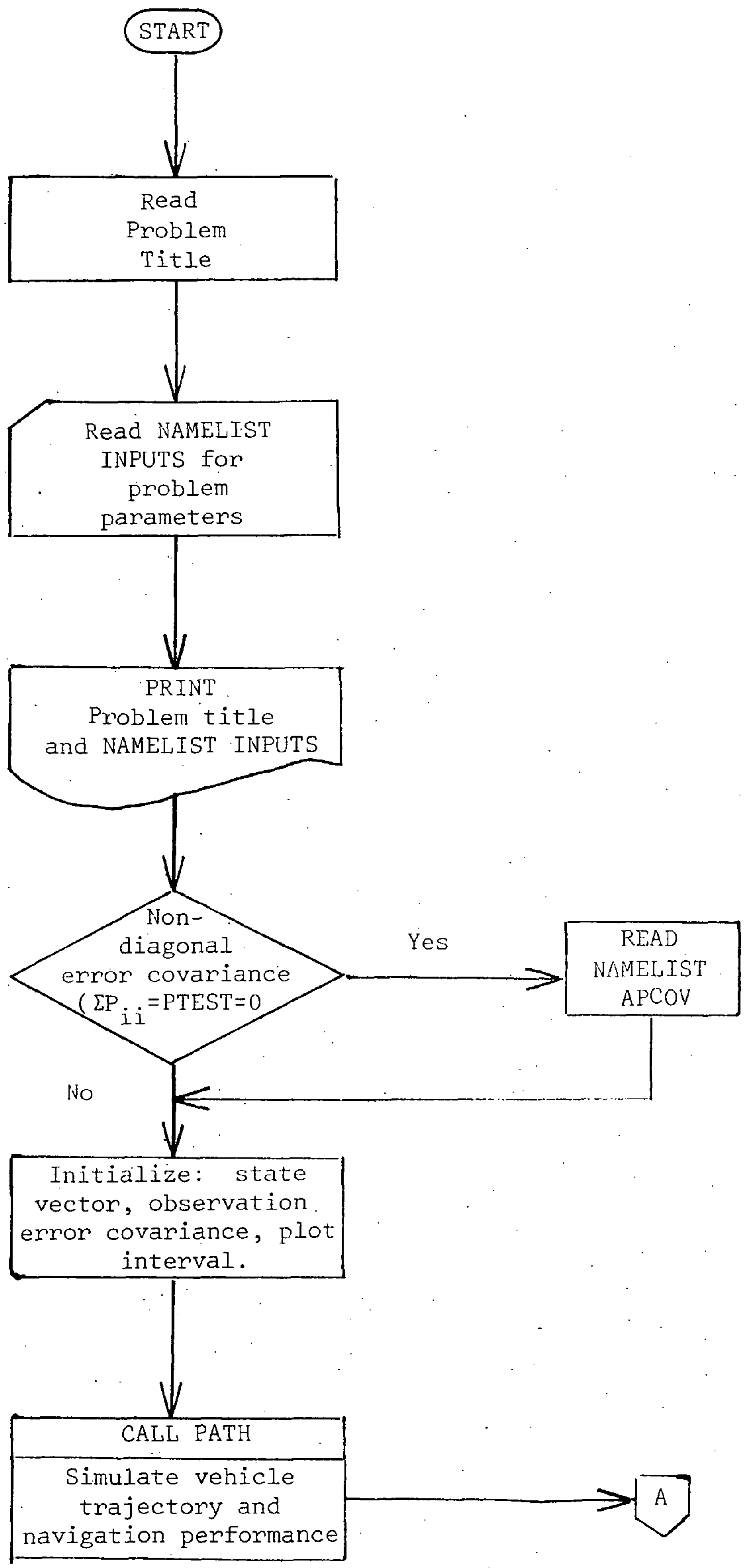

Figure 7. Main Program - LOGO

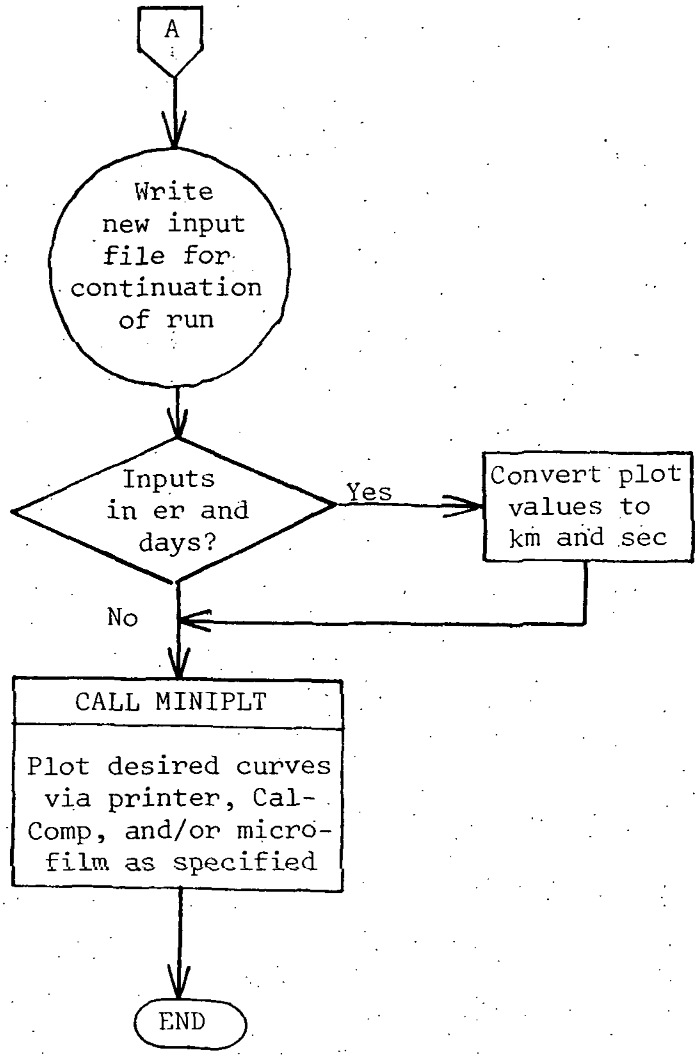

Figure 7. (Cont'd.)

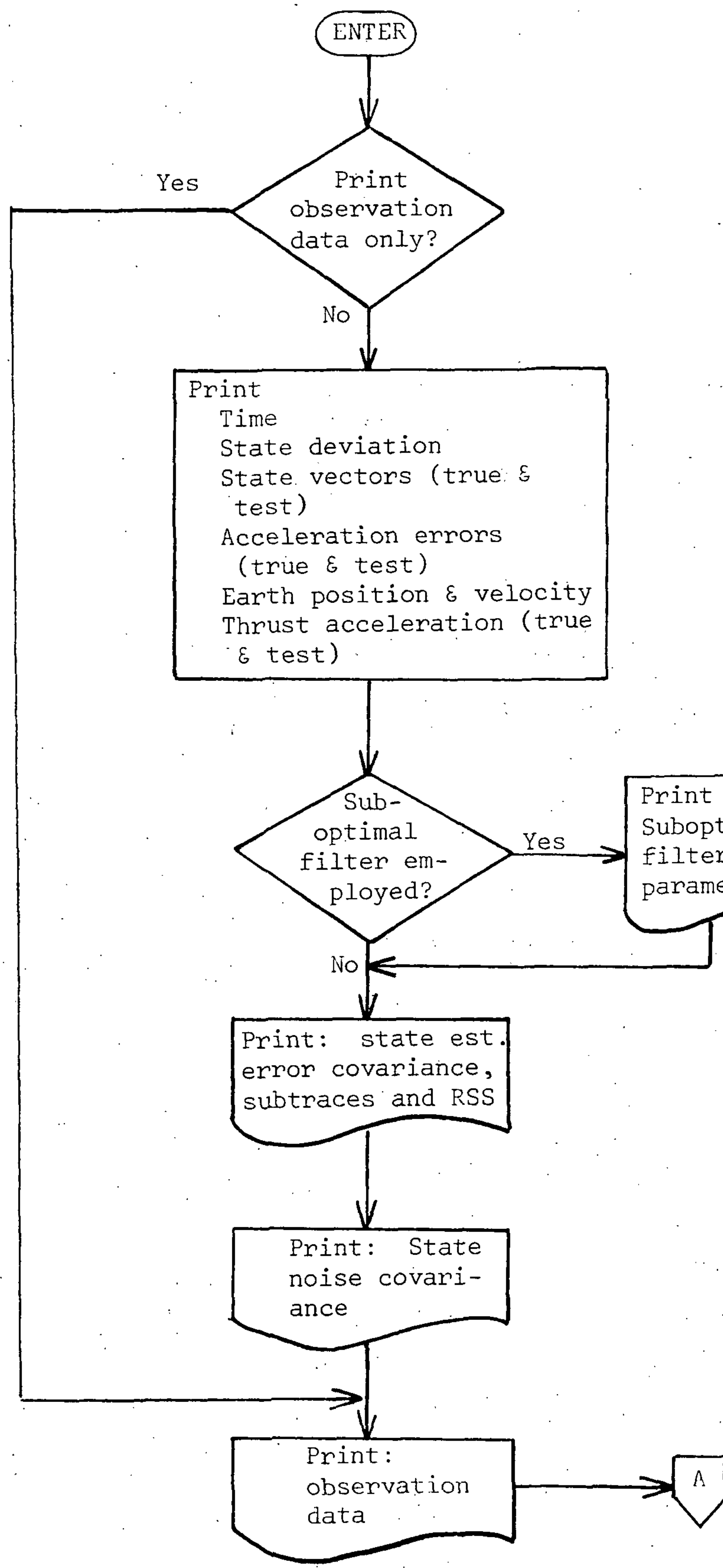

Figure 8. Subroutine OUTPUT

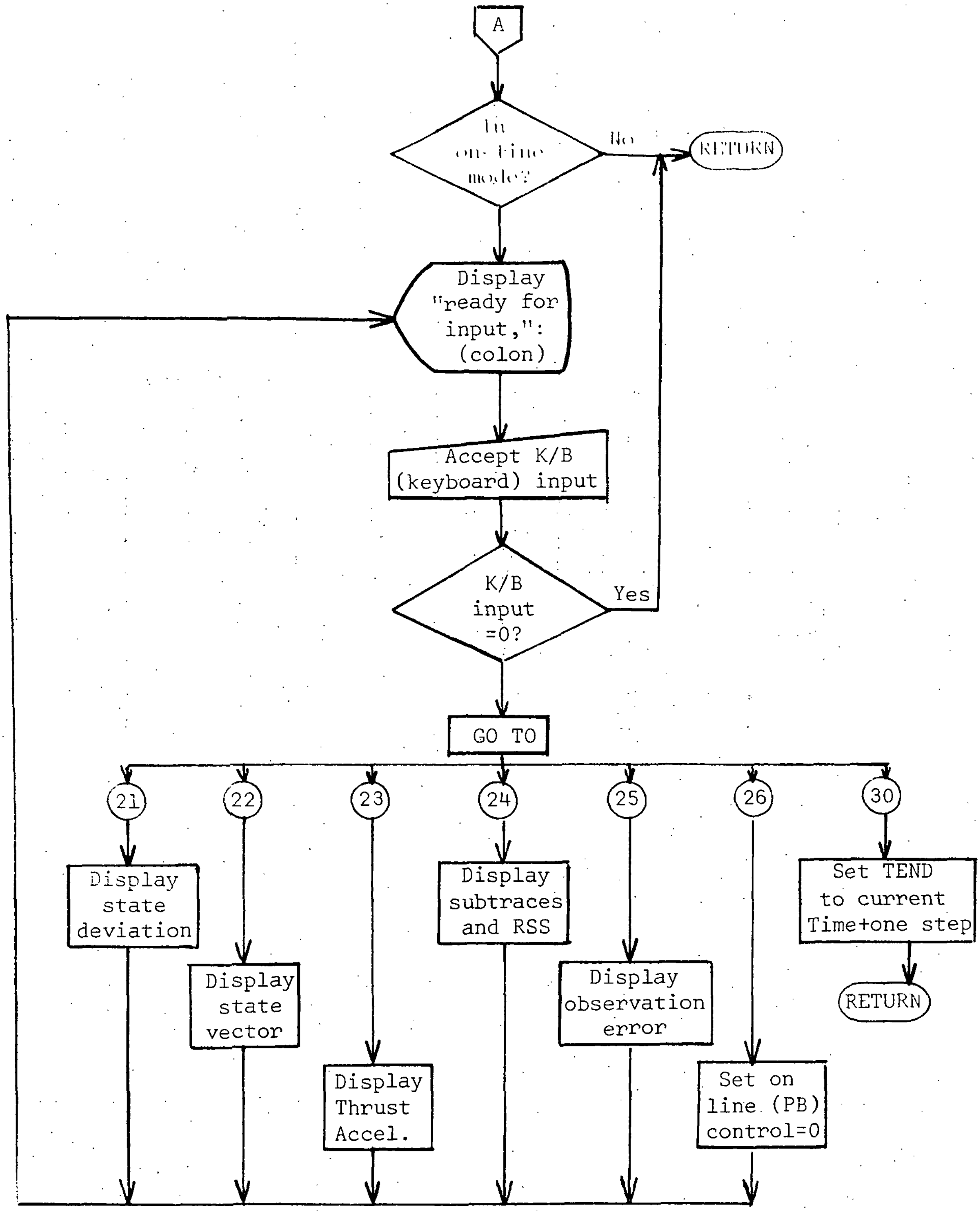

Figure 8. (Cont'd.)

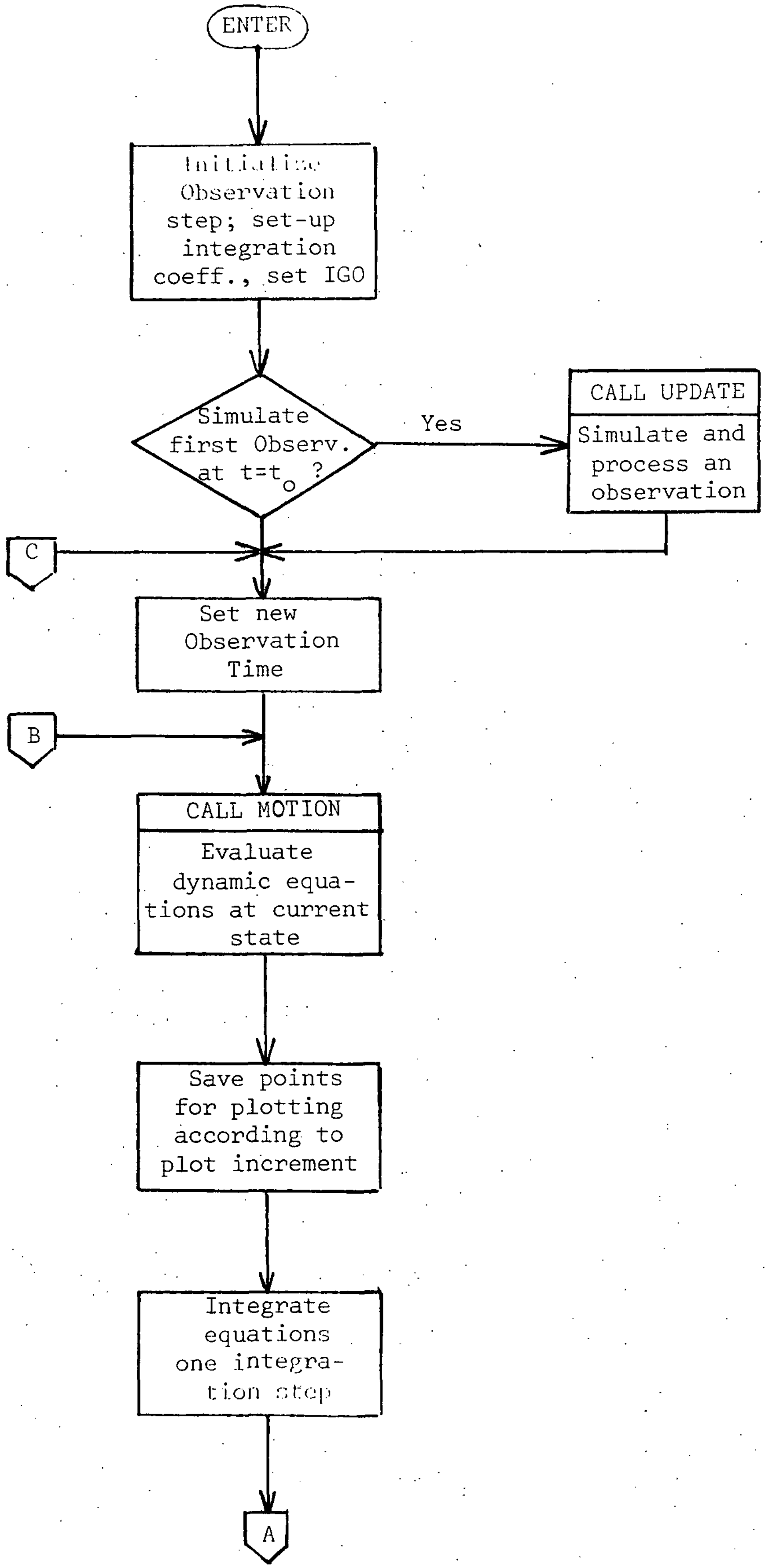

Figure 9. Subroutine PATH

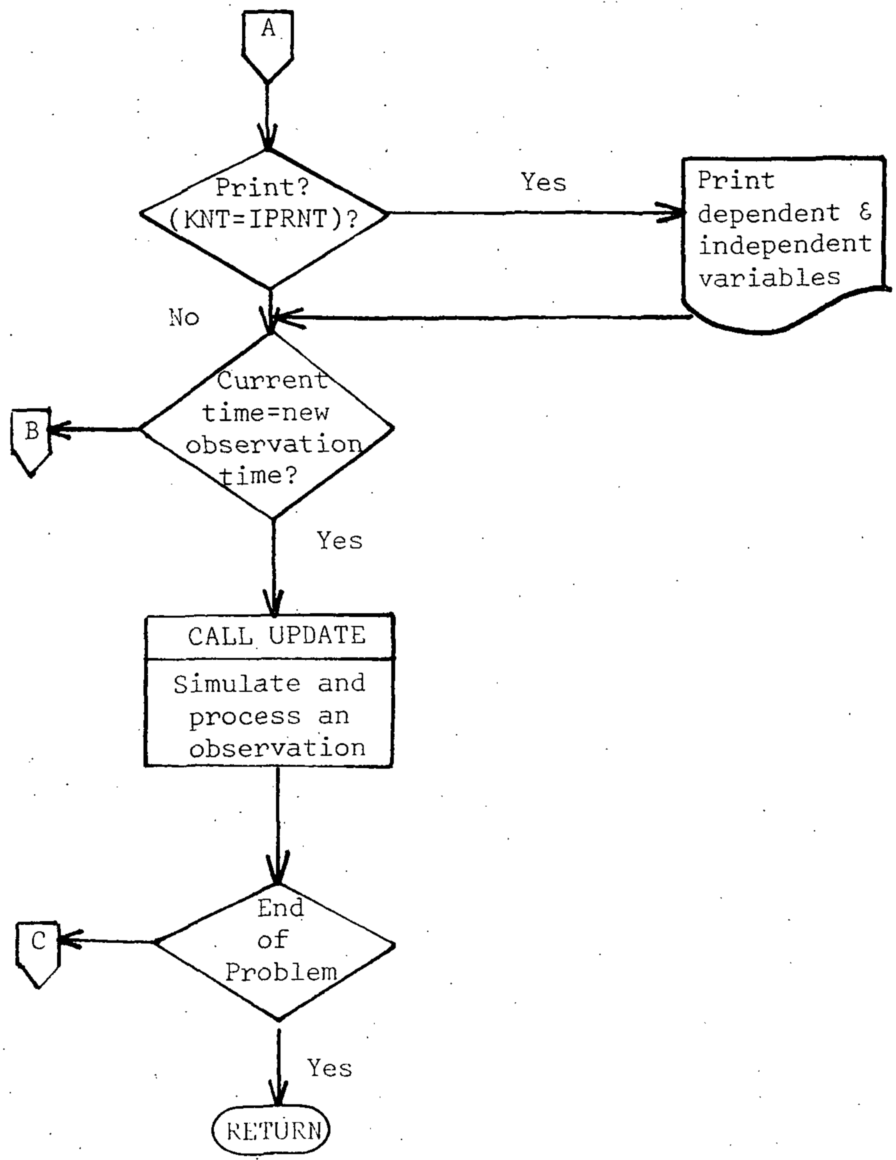

Figure 9. (Cont'd.)

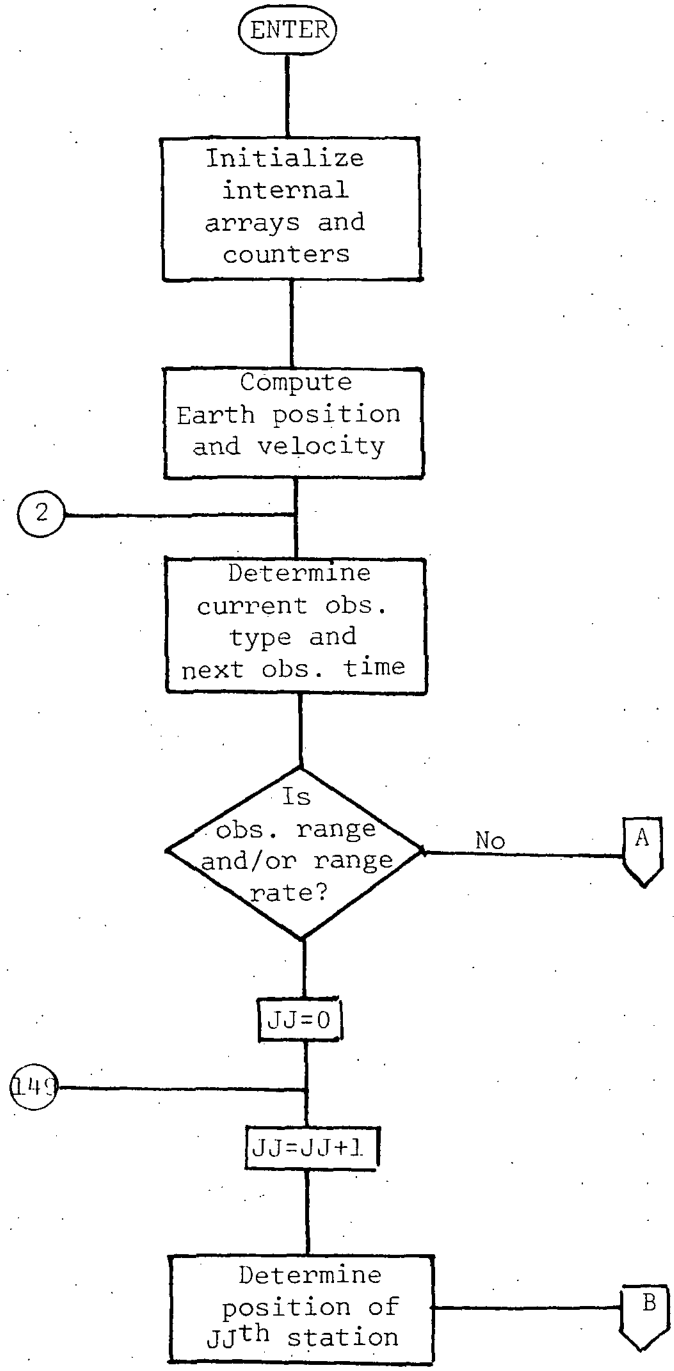

Figure 10. Subroutine UPDATE

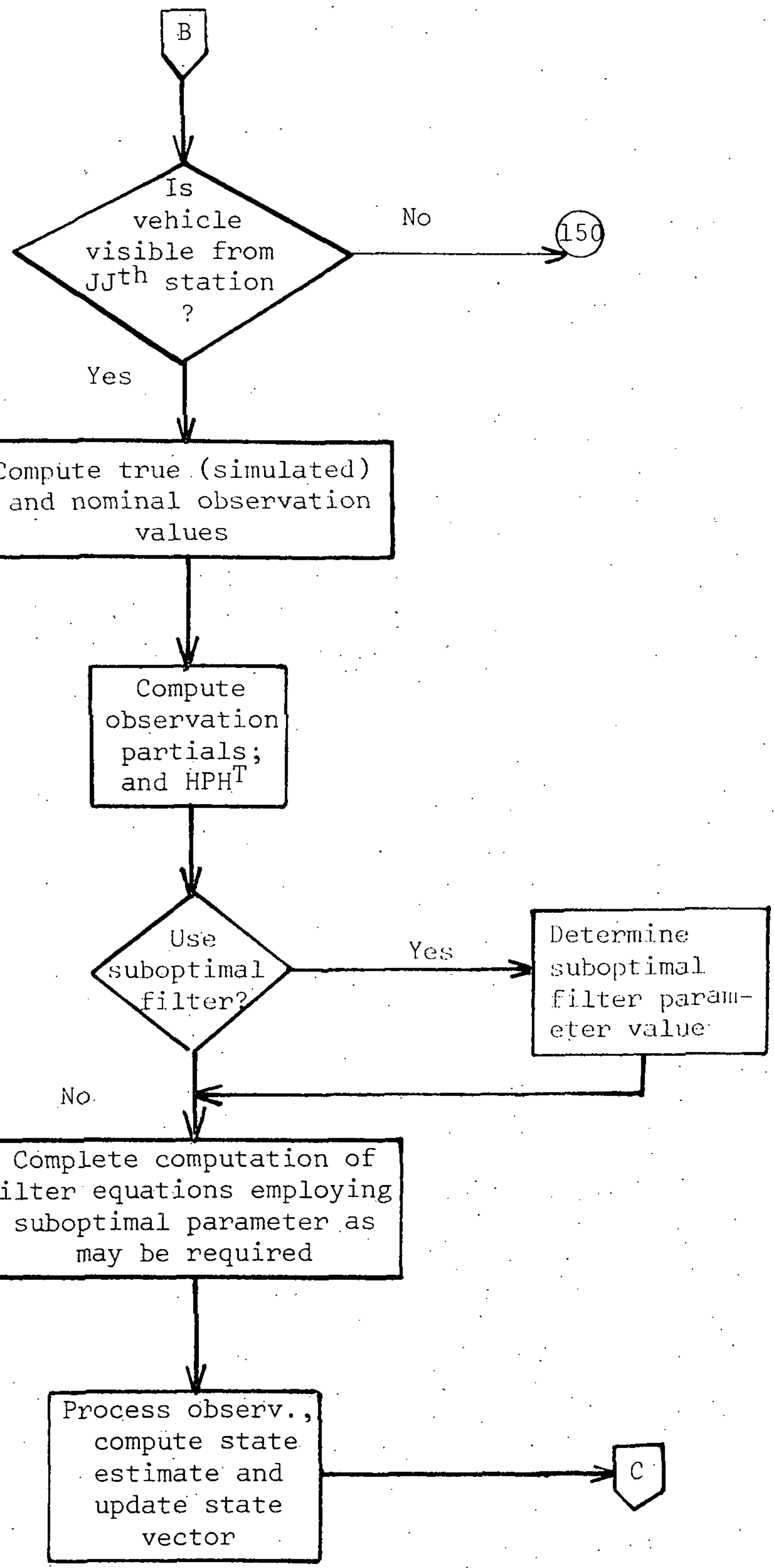

Figure 10. (Cont'd.)

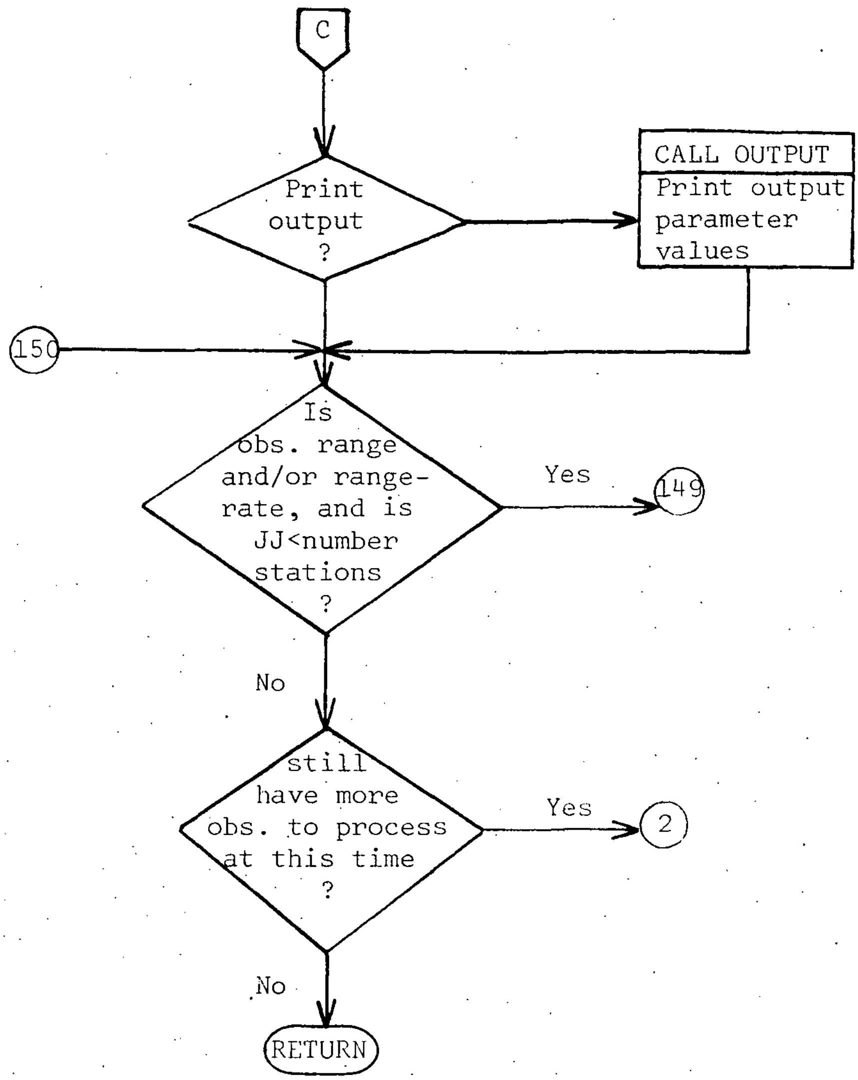

Figure 10. (Cont'd.)

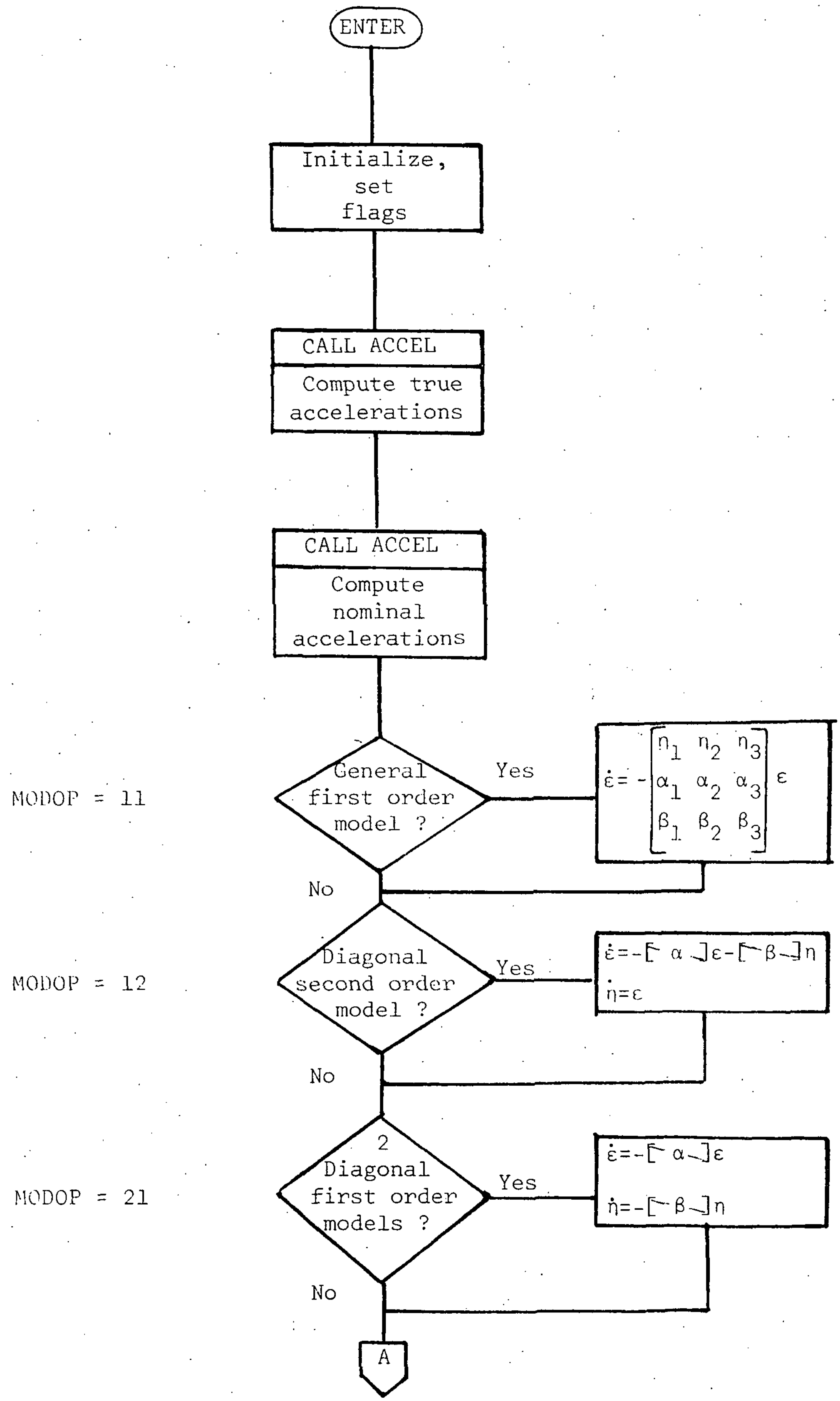

Figure 11. Subroutine MOTION

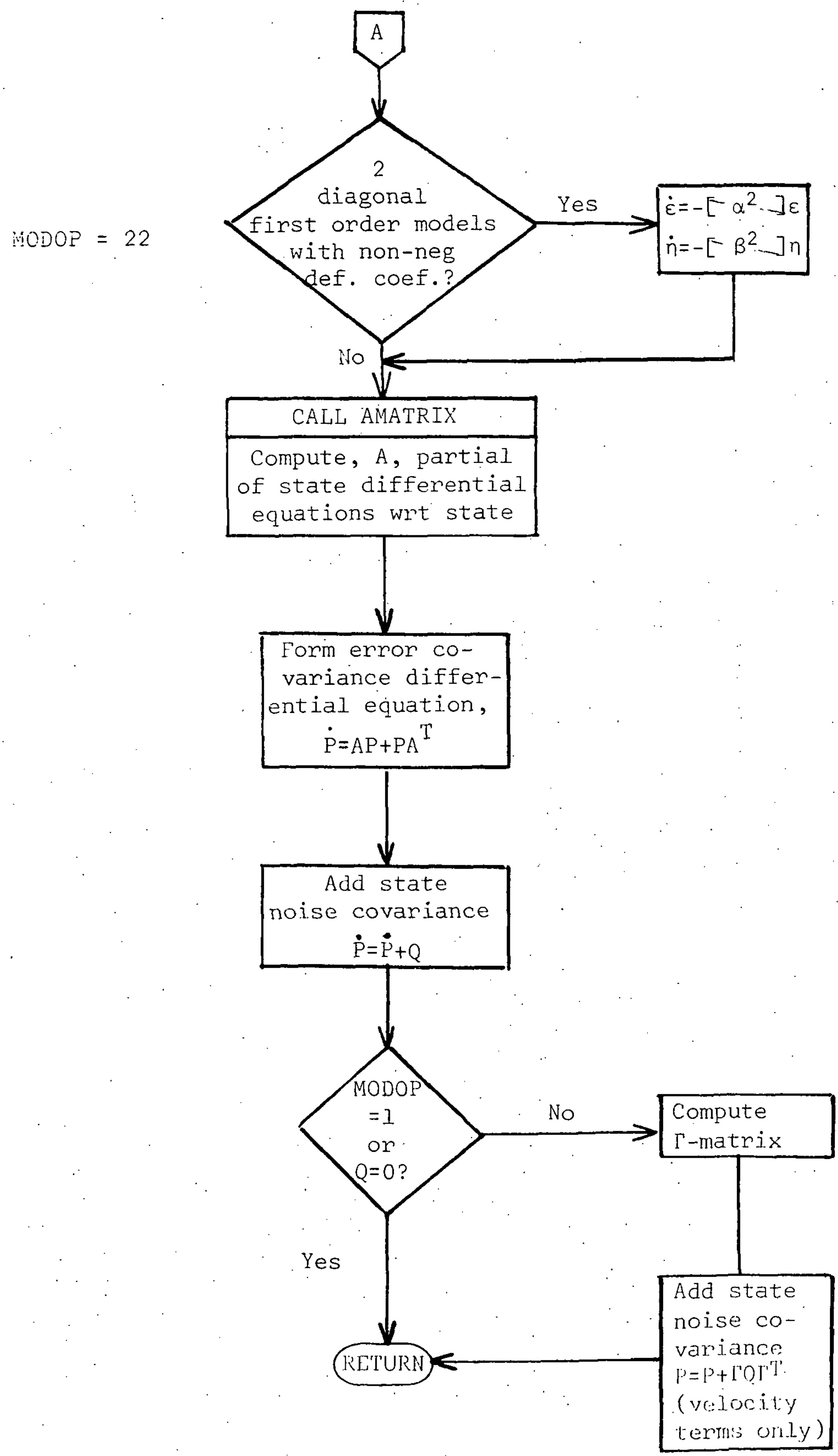

Figure 11. (Cont'd.)

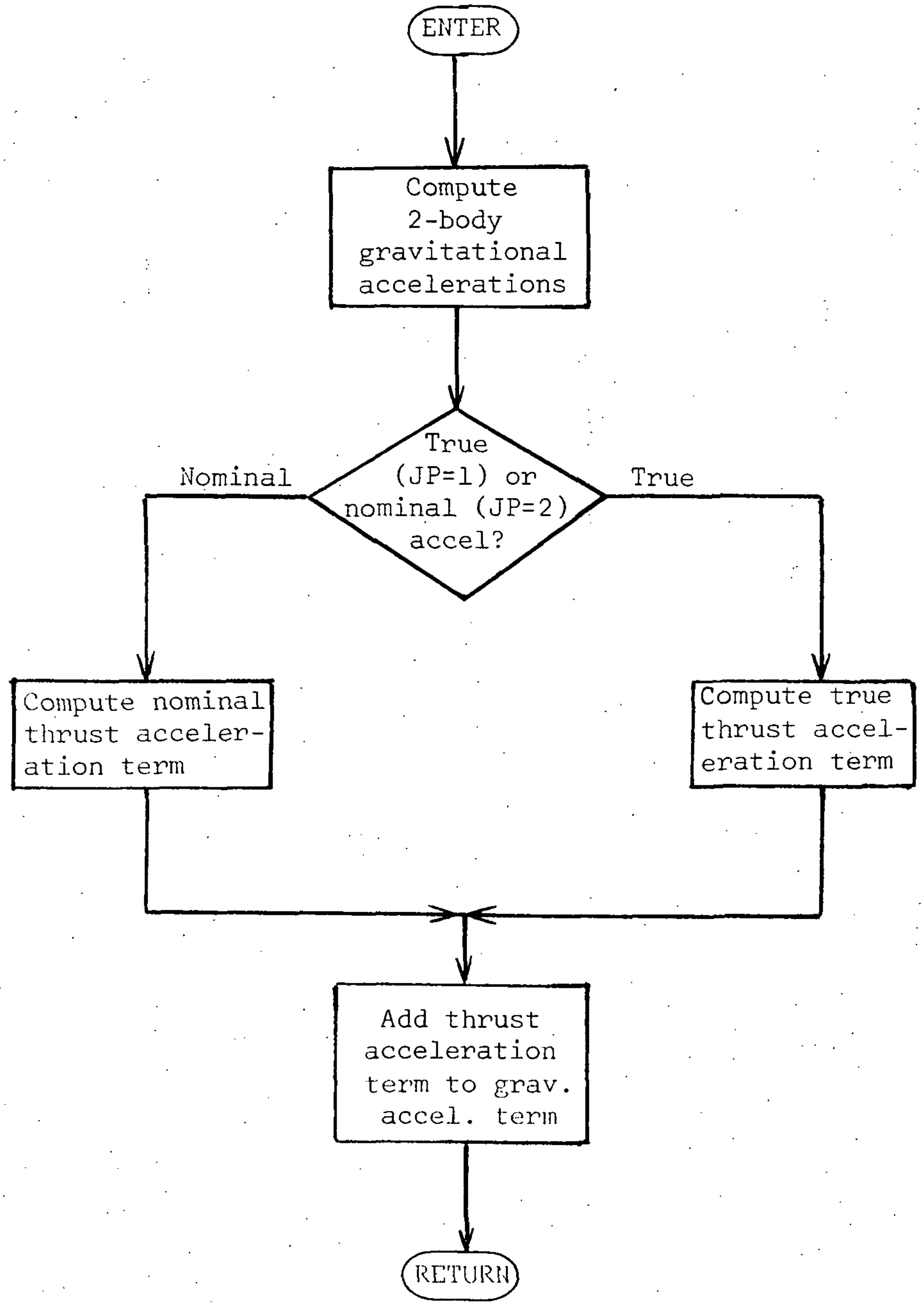

Figure 12. Subroutine ACCEL

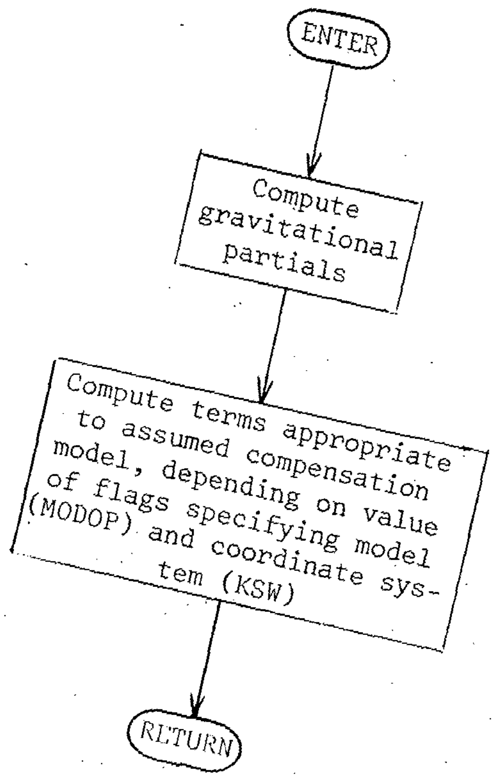

Figure 13. Subroutine AMATRIX

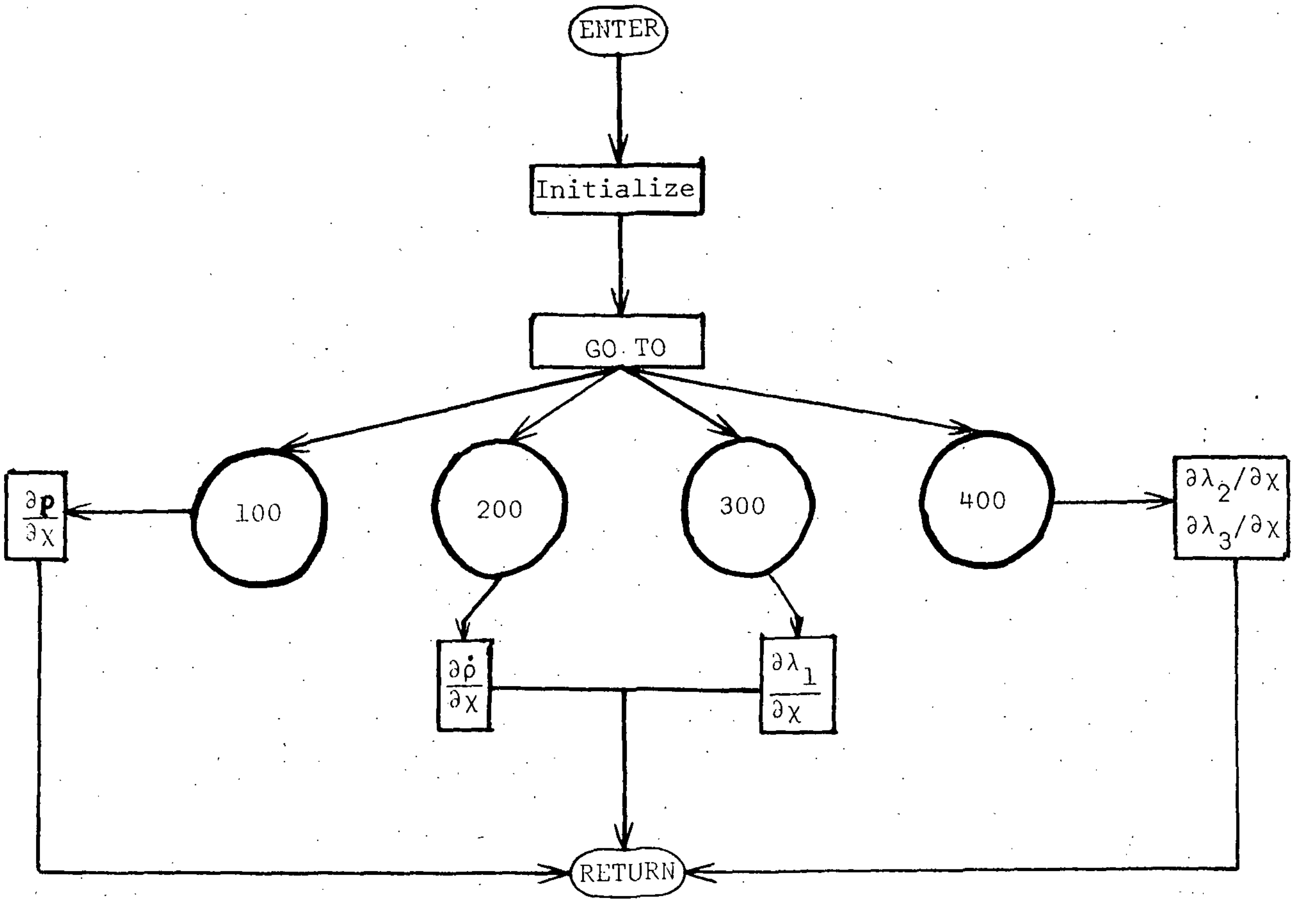

Figure 14. Subroutine HMATRIX

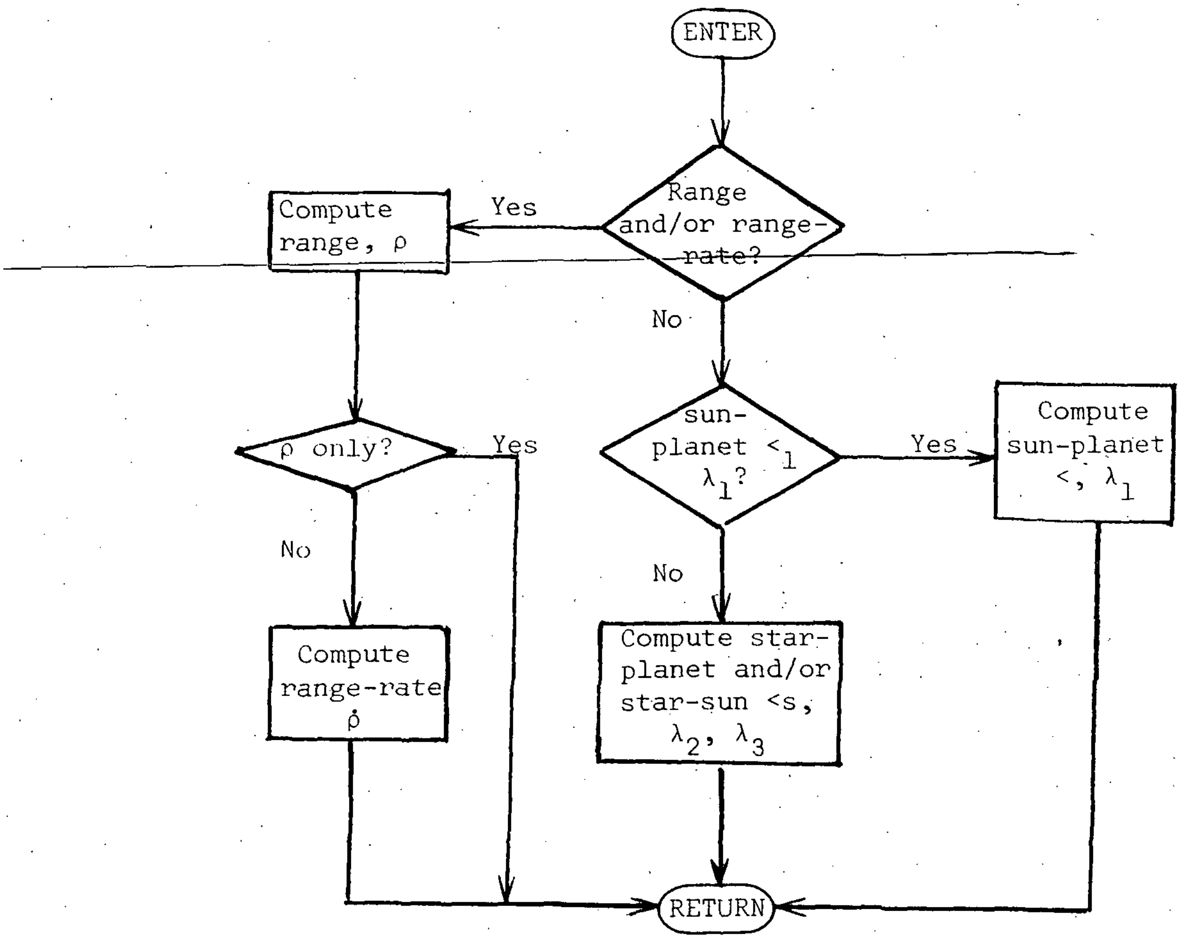

Figure 15. Subroutine OBSERV

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